

**Department of Physics**  
**Providence Women's College**

**E-Content of III Sem MSc Physics**

**Paper Name: Solid State Physics**

**Chapter: Superconductivity**

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6/9/21

## Superconductivity

- SC was discovered in 1911 by Heike Kamerlingh Onnes.
- Metals / material - Temp lowering - Resistance decreased.
- lattice will be opposing  $e^-$  flow through it
- lattice - regular arrangement of positive charged ions.
- lattice is in vibrational mode
- lattice -  $e^-$  interaction more  $\Rightarrow$  more (insulator) resistance
- lattice -  $e^-$  interaction <sup>weak</sup>  $\Rightarrow$  good conductor
- Temp  $\downarrow$  - lattice -  $e^-$  interaction <sub>weak</sub>  $\downarrow$  - Resistance  $\downarrow$ .
- In the case of mercury he observed sudden change in resistance observed at a particular temp.
  - temp below absolute zero  $\approx 4.18$  K.
- The resistance of mercury is coming to zero.
- This phenomena he named - Superconductivity.

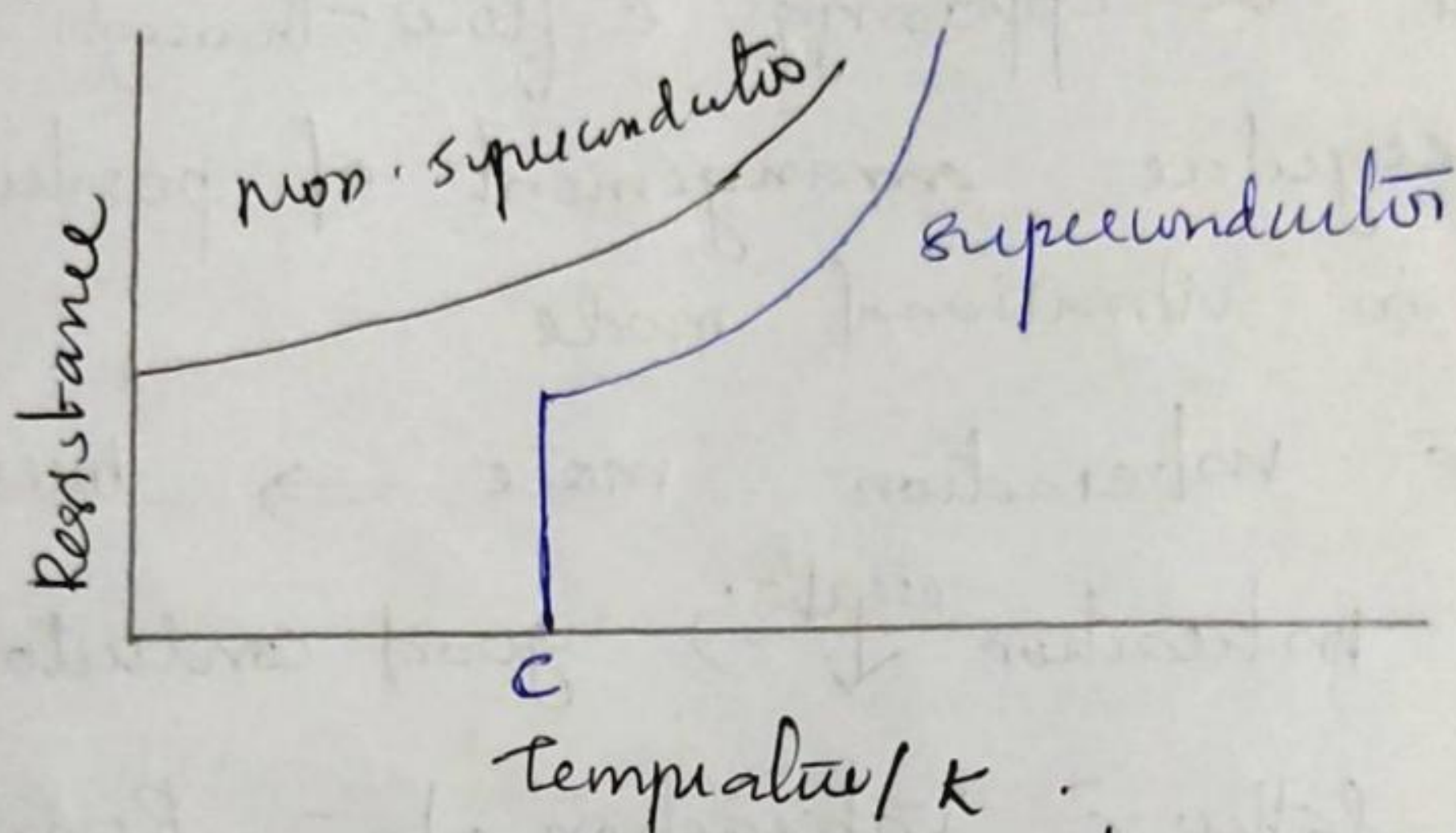
Superconductors are the material having almost zero resistivity and behave as diamagnetic below the superconducting transition temperature.

(diamagnetic material opposing applied electric field) mag

Superconductivity is the flow of electric current without resistance in certain metals, alloys, and ceramics at temperatures near absolute zero, and in some cases at temperatures hundreds of degrees above absolute zero  $= -273^\circ\text{K}$ . (reversible process - normal conductor to superconductor)



- As temperature decreases, a superconducting material's resistance gradually decreases until it reaches critical temperature. At this point resistance drops off, often to zero, as shown in the graph at right.



$C =$  critical temperature.

- Critical Temperature ( $T_c$ ) The temperature at which a material's electrical resistivity drops to absolute zero is called the critical temperature or transition temperature.

-  $T_c \propto C$  - function of impurity of the material.

- Below critical temp, material is said to be in superconducting and above this it is said to be in normal state. Below this temperature the superconductor also exhibits a variety of several astonishing magnetic and electrical properties.

Hg =  $T_c(K) = 4.15$

(adding impurities  $T_c$  changes)

### General Properties of Superconductors

- Electrical resistance: Virtual zero electrical resistance.
- Effect of impurities: when impurities are added to superconducting elements, the superconductivity is not lost.



but the  $T_c$  is lowered.

(impurities reduce the width of SC)  
line gradually decrease.

- Effects of pressure and stress: certain materials exhibit superconductivity on increasing the pressure in superconductors, the increase in stress results in increase of the  $T_c$  value.

### Effect of Magnetic Field

Critical Magnetic Field ( $H_c$ ) :- minimum magnetic field required to destroy the superconducting property at any temperature.

$$H_c = H_0 \left[ 1 - \left( \frac{T}{T_c} \right)^2 \right]$$

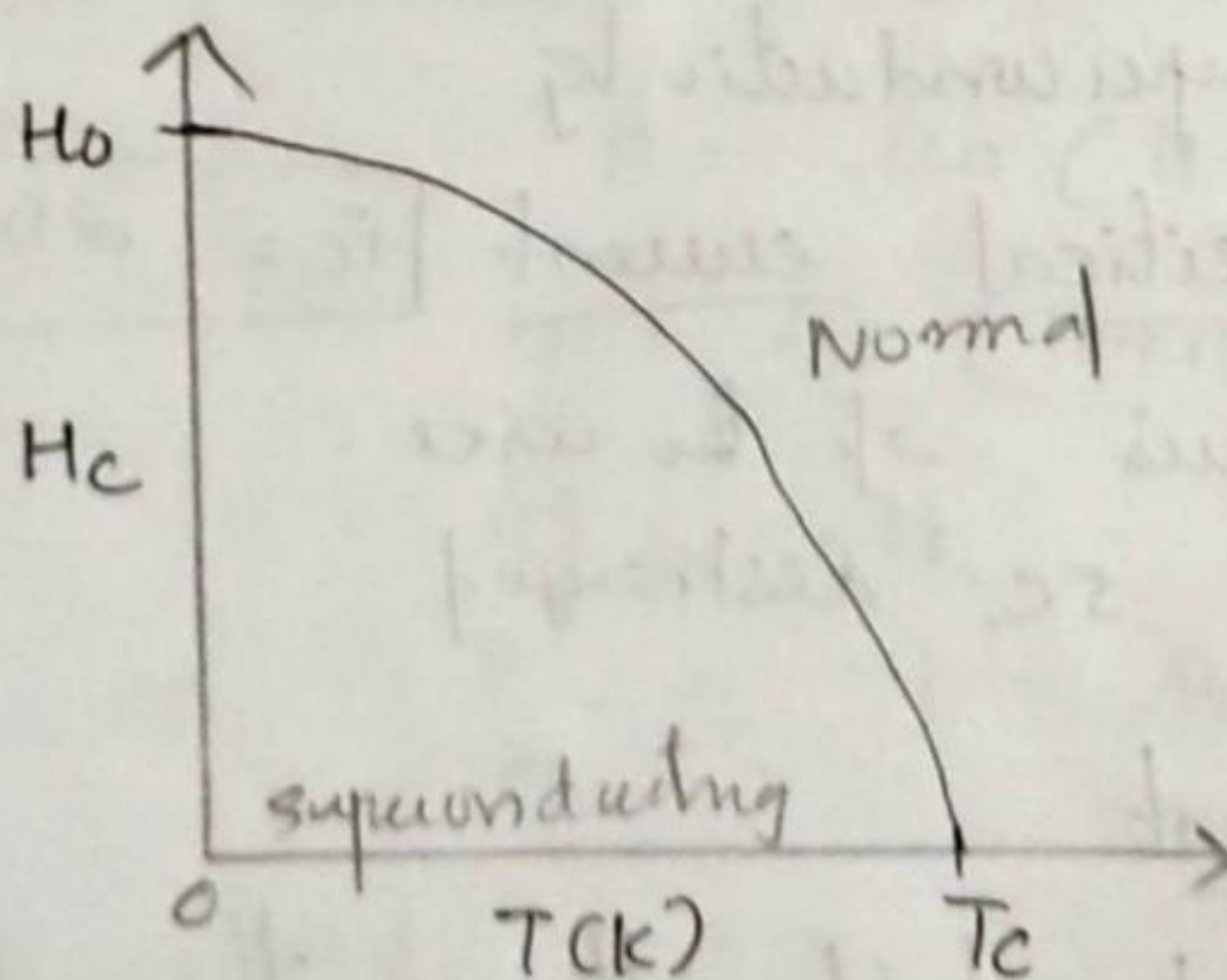
$H_0$  = critical field at 0K

$T$  = Temp below  $T_c$

$T_c$  = transition temperature.

element	$H_c$
Nb	198
Pb	80.3
Sn	30.9

$0 - T_c$   
max  $H$  applied to  
destroy SC  
after  $T_c$   
already normal  
conductor



- Superconductor showing diamagnetic property, superconductor always try to oppose the external magnetic field
- Superconductor will be cutting the external mag flux lines.
- Current will be induced, when flux is cut.
- That current will be flowing outside the SC. on the



- surface of superconducting material the current will be flowing - screening current
- due to screening current it will be producing a magnetic field.
  - Direction of this Mag field will be oppo to the direction of external mag. F.
  - If we keep on increasing ext mag field, screening current also will be increasing so the opposition also will increase.
  - After a certain point, the mag field started penetrating inside the body.
  - At a particular value of magnetic field the full flux lines will be started penetrating inside the body.
  - The S.C property can be destroyed.

### Effect of Electric Current

- large electric current - induces magnetic field - destroys superconductivity

- Induced Critical current  $i_c = 2.17 r H_c$

$r =$  radius of the wire.

if  $i > i_c$  - SC destroyed

### Persistent Current

- steady current which flows through a superconducting ring without any decrease in strength even after the removal of the field.
- diamagnetic property.



# Meissner effect super conductor is not ideal conductor

- when superconducting material cooled below its  $T_c$  it becomes resistanceless & perfect diamagnetic.

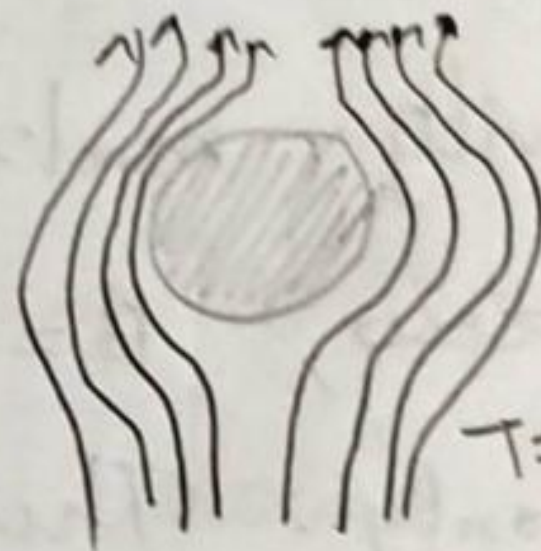
(induced mag field  $B=0$  inside material).

when superconductor placed inside a magnetic field  $\mu_0 H$   $T < T_c$  all magnetic flux is expelled out of it the effect is called Meissner effect.

- Perfect diamagnetism arises from some special magnetic property of superconductor.



$T > T_c$



$T = T_c$

$B=0$

- If there is no magnetic field inside the superconductor relative permeability or diamagnetic constant  $\mu_r = 0$

- total magnetic induction  $B$  is zero.

$\Rightarrow B = 0 \Rightarrow B = \mu_0 (H + M)$

$0 = \mu_0 (H + M)$

$M = -H$

(Perfect diamagnetic material)

$\therefore \frac{M}{H} = -1 = \chi_m$

magnetic super susceptibility

$H =$  applied field

$M =$  magnetization

induced

$M > H$

= Ideal case - Temp  $\downarrow$  Resistance  $\downarrow$  each zero value

- superconductor obeys the ideal case.

Superconductors - ideal case or not?

diamagnetism is essential property of SC but all diams are not SCs.



Meissner effect = zero resistivity?

$$B=0 \Rightarrow \rho=0?$$

For zero resistivity to occur  $B=0$ , not need.

$B=0$  does not follow from zero resistivity ( $\rho=0$ )

from ohm's law  $J = \sigma E$

$$J = \sigma E \\ \rho = \frac{E}{J}$$

$$E = \rho J$$

If  $\rho \rightarrow 0$ ,  $J$  is finite, current density  $\Rightarrow E = 0$ .

maxwell's eqs equalator  $\Rightarrow \nabla \times E = -\frac{\partial B}{\partial t}$

$$\Rightarrow \frac{\partial B}{\partial t} = 0$$

or  $B$  is constant, so  $B \neq 0$  always.

for a zero resistivity material magnetic induction is not necessarily zero;  $B=0$  is a special property of superconductor only. This strong repulsion of external magnetic field is called Levitation Effect.

- (strongly opposing mag F., mag F levitates above S.C.).

TP  $\rightarrow$  SC - zero resistivity & Meissner effect, - cannot explained by Maxwell eqs & Ohm's law.

### London Equation

- two fluid eqs.

- According to London's theory there are two types of electrons in SC

- Super electrons & Normal electrons  
(at 0°K) (at  $T_c$ )

- At 0°K there are only Super electrons.



- with increasing temp. Super electrons ↓  
Normal electrons ↑.

- Normal  $\bar{e}$  have resistance = lattice vibrations  
- Super  $\bar{e}$  - 'NO' resistance

- The current carried by super  $\bar{e}$   $\gg$  current carried by normal  $\bar{e}$ .

$n_n$  = number density of normal  $\bar{e}$

$u_n$  = drift velocity of normal  $\bar{e}$

$n_s$  = no. density of super  $\bar{e}$

$u_s$  = drift velocity of super  $\bar{e}$

Equation of motion of Super electron under electric field is  $m \frac{du_s}{dt} = -eE$  ( $ma = qE = F$ )

(contribution of due to super  $\bar{e}$   $\gg$  cont. due to normal  $\bar{e}$ )

Now current & drift velocity are related as

$I_s = (-) n_s e A u_s$   $A = a^2$

$J_s = (-) n_s e u_s$

$\therefore u_s = \frac{-\vec{J}_s}{n_s e}$

$m \frac{d}{dt} \left( \frac{-\vec{J}_s}{n_s e} \right) = -e \vec{E}$

$\therefore \frac{d\vec{J}_s}{dt} = \frac{n_s e^2 E}{m}$

London's first equation.  
prove zero resistivity

If  $E = 0$   $\frac{d\vec{J}_s}{dt} = 0 \Rightarrow J = \text{constant}$  (finite)  
current density constant

without any (external) electric field, there will be persistent current will be flowing.



$$\vec{F} = \frac{d\vec{J}_s}{dt} \frac{m}{n_s e^2}$$

$$E = -\frac{dA}{dt} \text{ potential}$$

$$\frac{d\vec{J}_s}{dt} \cdot \frac{m}{n_s e^2} = -\frac{d\vec{A}}{dt}$$

$$\frac{d}{dt} \left( \frac{m}{n_s e^2} \vec{J}_s \right) = -\frac{d\vec{A}}{dt}$$

$$\therefore \frac{m}{n_s e^2} \vec{J}_s = -\vec{A}$$

$$\vec{J}_s = -\frac{n_s e^2 \vec{A}}{m}$$

London's second equation

Topdown

- T.P meissner effect.

Again from Ampere's law,

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}_s$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \left( -\frac{n_s e^2}{m} \vec{A} \right)$$

Take curl on both sides.

$$\vec{\nabla} \times \vec{\nabla} \times \vec{B} = \mu_0 \left( -\frac{n_s e^2}{m} \vec{\nabla} \times \vec{A} \right)$$

Now

$$\vec{\nabla} \times \vec{\nabla} \times \vec{B} = \vec{\nabla} \times \vec{\nabla} \cdot \vec{B} - \nabla^2 \vec{B} \quad \& \quad \vec{\nabla} \times \vec{A} = \vec{B}$$

$$\vec{\nabla} \cdot \vec{\nabla} \cdot \vec{B} - \nabla^2 \vec{B} = \mu_0 \left( -\frac{n_s e^2}{m} \vec{B} \right)$$

$$\text{So } \vec{\nabla} \cdot \vec{\nabla} \cdot \vec{B} = (\vec{\nabla} \cdot \vec{\nabla} \cdot \vec{\nabla} \times \vec{A}) = 0$$

$$\therefore \nabla^2 \vec{B} = \frac{\mu_0 n_s e^2}{m} \vec{B}$$

$$\text{Assume } \mu_0 \frac{n_s e^2}{m} = \frac{1}{\lambda^2} = \frac{\mu_0 n_s e^2}{m}$$

$$\nabla^2 \vec{B} = \frac{1}{\lambda^2} \vec{B}$$



$$\nabla^2 \vec{B} - \frac{1}{\lambda^2} \vec{B} = 0$$

$\lambda$  is called London penetration length.

from London's eq<sup>n</sup>

$$B(x) = B_0 e^{-x/\lambda_L}$$

(-ve sign indicates decreasing)

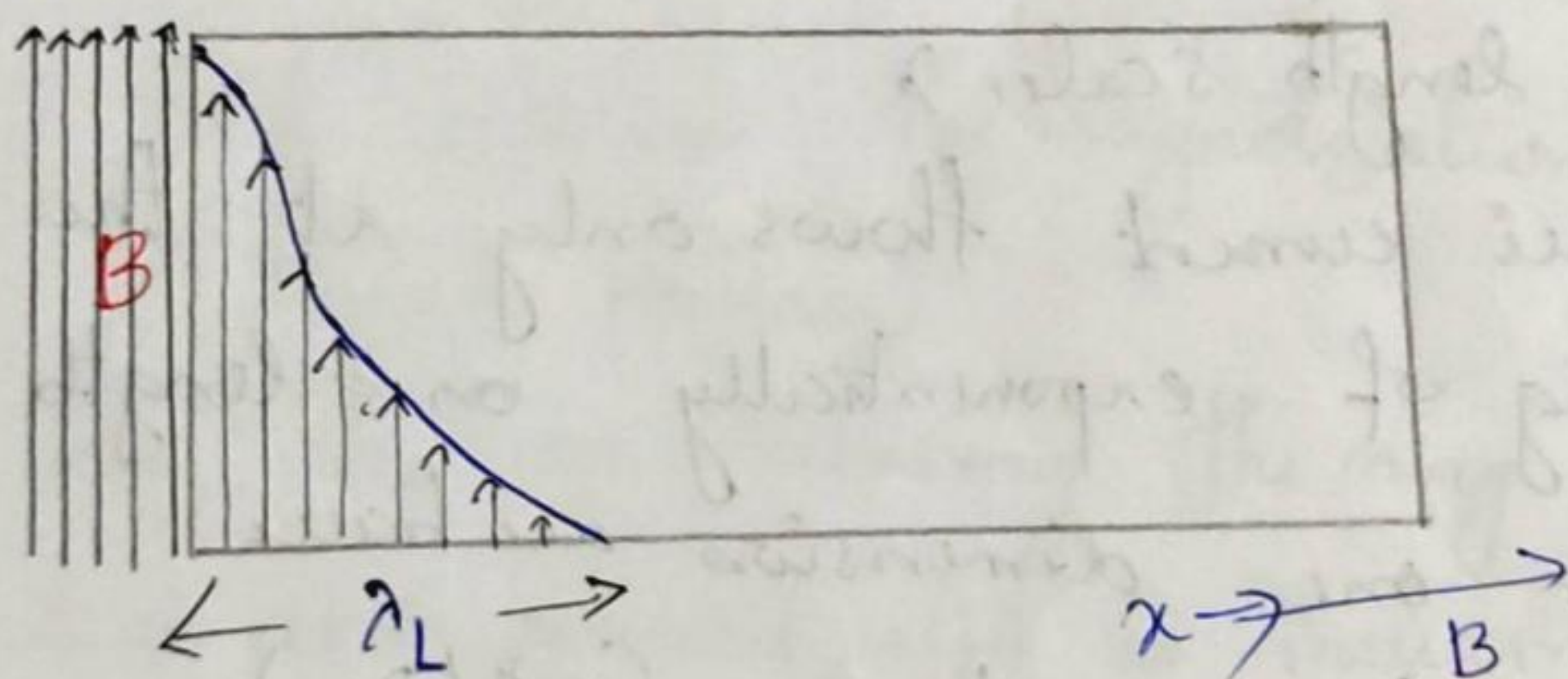
Penetration depth is the distance upto which magnetic lines penetrate through the material, when placed in a magnetic field.

$$\lambda_L = \text{penetration depth} = (m/nq^2e_0)^{1/2}$$

(considering x direction)

$B(x)$  = mag field at any position  $x$

$B(0)$  = mag field at  $x=0$  (starting point)



coherence length:

where coherence length is the range in a superconductor in which superconducting electrons remain in the same state in a spatially varying magnetic field.

The resistivity of the superconductor suddenly falls to zero indicates that all the electrons in the material come to the same state suddenly ( $10^{-4}$  cm) a long range order.



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superconductivity can be destroyed by

- increasing temp
- increasing magnetic field
- increasing current.

### Summary of London equations

The London's produced a phenomenological model of superconductivity which provided equations which described but did not explain superconductivity.

Starting with with the observation that superconductors expel all magnetic flux from their interior, they demonstrated the concept of the penetration depth; showing that

- flux does penetrate, but falls off exponentially on a length scale,  $\lambda$ .

- electric current flows only at the surface, again falling off exponentially on a length scale,  $\lambda$ .

So, in just one dimension we have

$$B(x) = B_A \exp(-x/\lambda_L)$$

$$J_y(x) = J_A \exp(-x/\lambda_L)$$

with  $\lambda_L = \sqrt{m/\mu_0 n_s e^2}$  London penetration depth

$\lambda_L$  is very small  $\Rightarrow$  flux will be penetrating on the surface only over a small thickness.

Inside the material flux = 0.

(If the thickness of the film is comparable to the penetration depth, you can observe that inside the superconducting material there will be flux. This is not obeying the

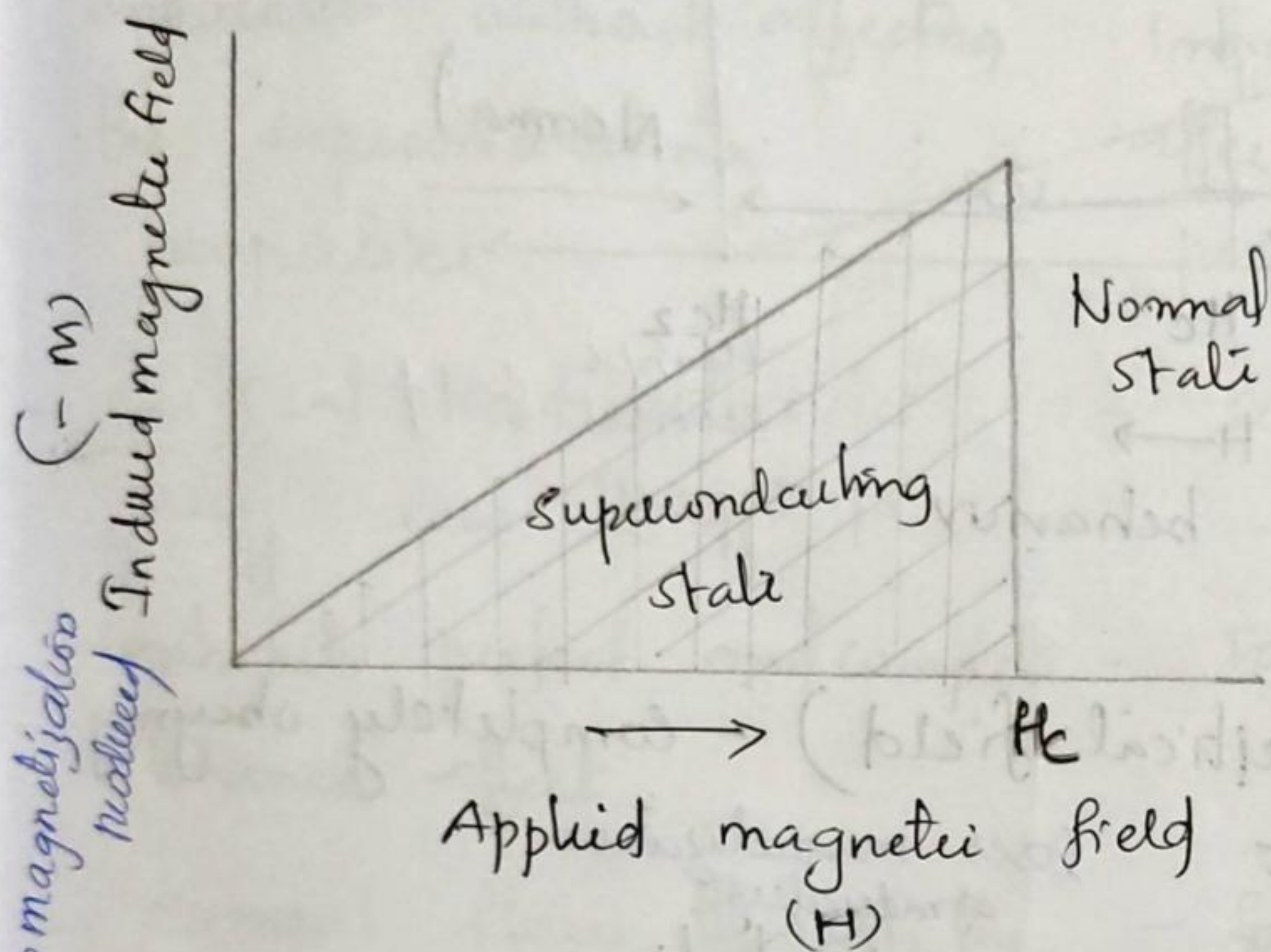


meissner effect)

On the basis of meissner effect, SCs can be classified into two

- 1) Type I
- 2) Type II

Type 1



- As the field increases the magnetization also will increase (linear increase).

- If you keep on increasing the mag field the magnetization ~~also~~ induced will be increasing in the opposite manner.

- magnetization is induced to the surface current (screening current)

- the induced (opposite) mag field and Applied mag field are equal up to the critical field  $H_c$ .

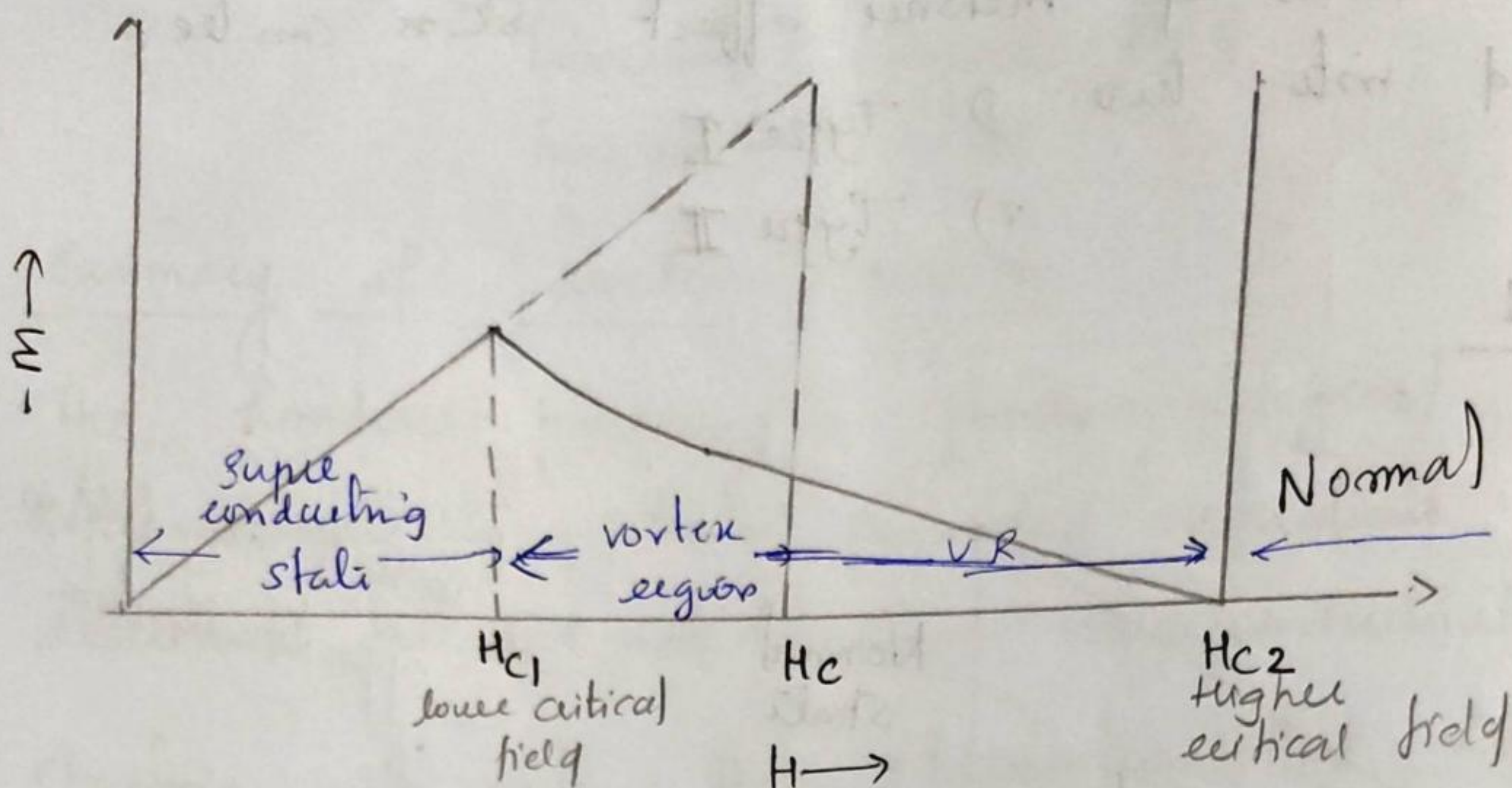
- above  $H_c$  = Normal state.

- upto  $H_c$  = obeys meissner effect - (no flux inside the material).

- magnetic field produced by the superconducting material are not allowing the external field to enter inside the body.



## Type 2



- Solid line represent behavior of Type II
- dotted line Type I
- upto  $H_{c1}$  (lower critical field) - completely obeying

the meissner effect loses magnetization gradually

between  $H_{c1}$  &  $H_{c2}$  - obeying mixed state

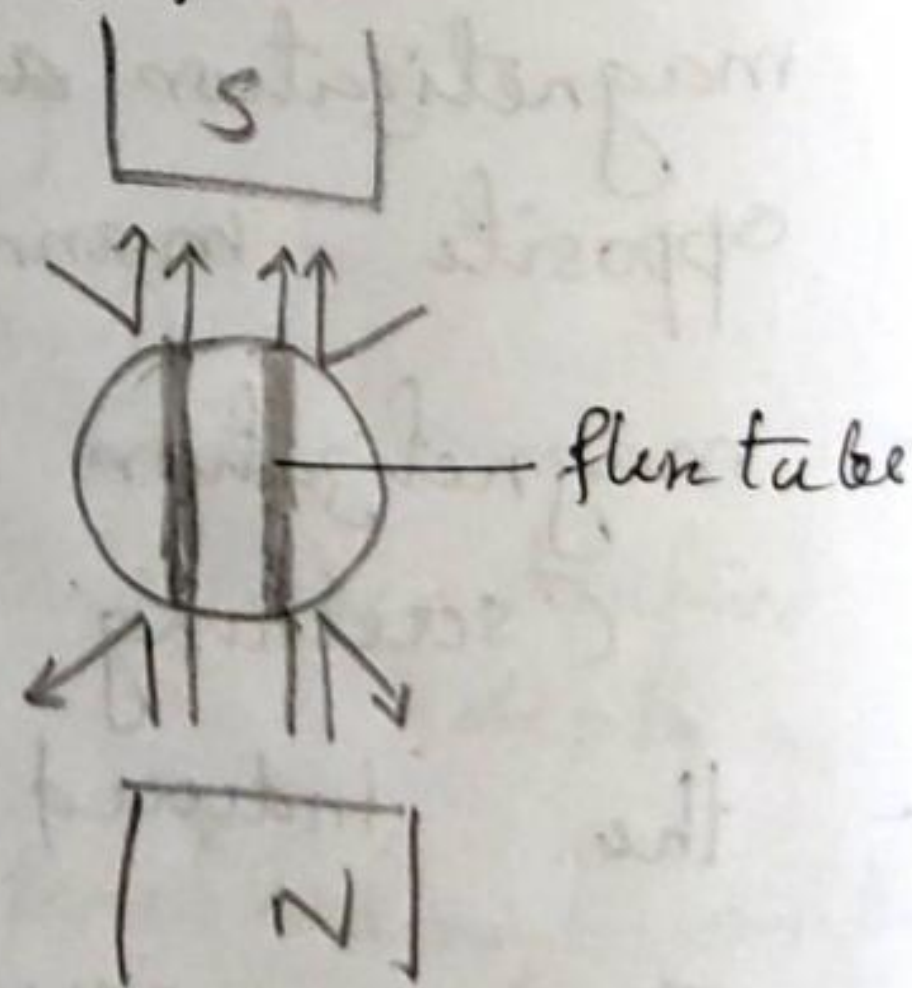
- flux will be penetrating through the body, but as well as behaving like a superconductor

## Type 1



- do not allow mag fields to penetrate.
- Above  $H_c$  - lose their superconducting properties.

## Type 2



- allow magnetic fields to penetrate b/w  $H_{c1}$  &  $H_{c2}$ .
- lose their SC properties above  $H_{c2}$ .
- flux tubes :- a normal non superconducting regions in a



Type II S.C. vs b/w  $H_{c1}$  &  $H_{c2}$ .

### Type 1

- Soft Superconductors are those which can tolerate impurities without affecting the superconducting properties

- critical field value is very low.

- exhibits perfect and complete Meissner effect

- current flows through the surface only

- These materials have limited technical applications because of very low field strength values

(0.2 Tesla)

### Type 2

- Hard Superconductors are those which cannot tolerate impurities, i.e., the impurities affect the superconducting property.

- critical field value is very high

- Don't exhibit perfect and complete Meissner effect

- found that current flows throughout the material.

- These materials have wide technology of very high field strength value.

- carry high current density (40 Tesla)

### Isotope effect

- evidence of S.C. effect = isotope effect & energy gap.

- observed by Maxwell in 1950.

- critical temp of SC varies with isotopic mass.

- (Isotopes - same no. of p & e, neutron number changing

- Z number not changing, lattice not changing  
- n change - material behaviour change.)



- Transition temp of mercury changes from 4.185 K to 4.146 K as the isotopic mass  $M$  varies from 199.5 to 203.4.

-  $T_c$  varies with the isotopic mass  $M$  as

$$T_c \propto M^{-\alpha} \quad \alpha \approx \frac{1}{2}$$

$$T_c M^{1/2} = \text{constant}$$

- The isotopic mass can enter in the process of the formation of the superconducting phase of the electron states only through the electron-phonon interaction.  
(phonon - lattice vibrational wave)

### Energy Gap.

[The energy gap b/w super  $\bar{e}$  and normal  $\bar{e}$  or  
- The energy needed to convert super  $\bar{e}$  to normal  $\bar{e}$ ]

-  $E_G$  not a fixed value.  $E_G$  chang w.r.t temp

- At  $T=0$  - gap will be maximum - only s-electrons.

- When  $T \uparrow$ , gap  $\downarrow$ ,  $T = T_c = E_G = 0$

- In SC  $E_G$  attached with Fermi level.]

- The energy gap in superconductors are attached to the Fermi gas.

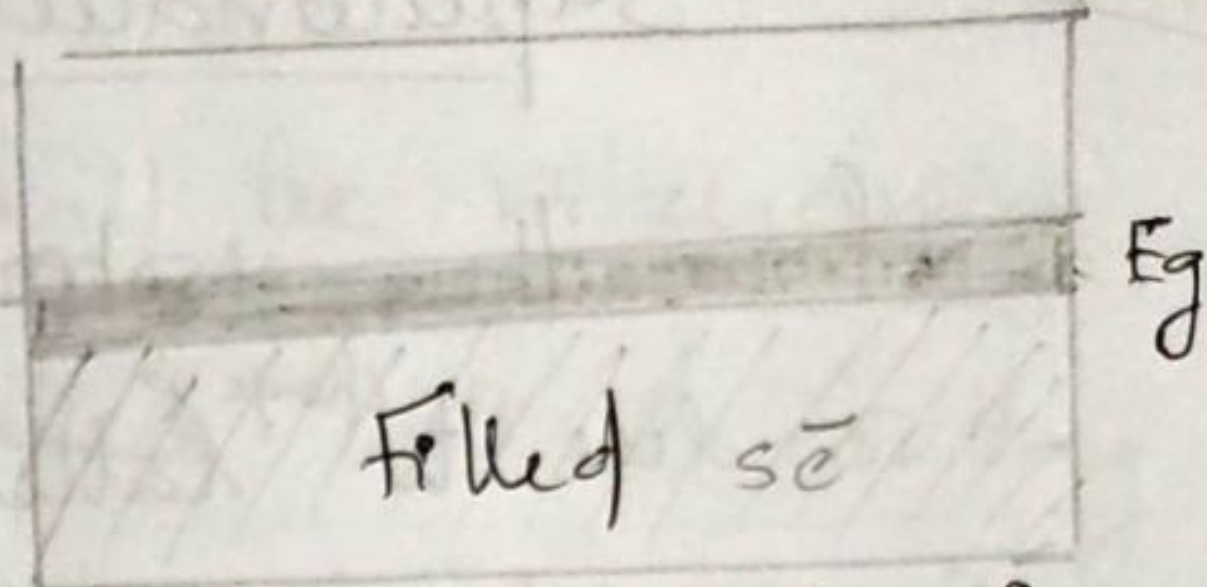
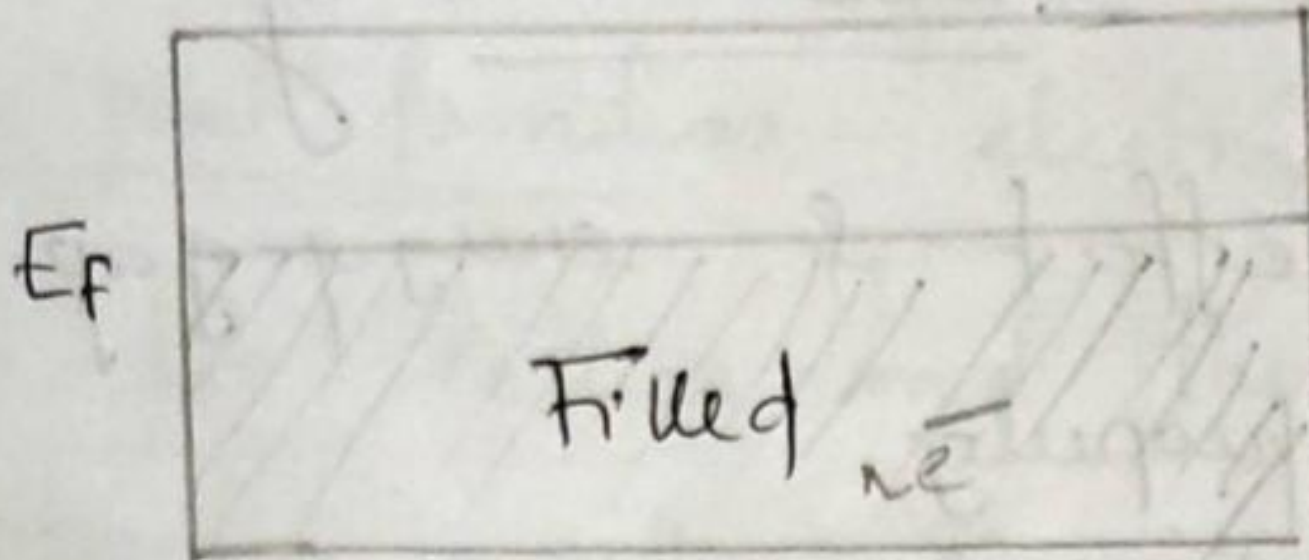
- Current flows despite the presence of a gap (current flows due to super  $\bar{e}$ ).

- The energy gap has no effect upon the behaviour of the special electrons that carry current in SC.

- (There will be a gap b/w filled state & unfilled state -  $E_G$ )



At  $T=0K$



- The ~~energy gap~~ conduction band is normal state

Energy gap at the fermi level is the superconducting state.

- The energy gap varies with temperature Unlike insulators and semiconductors.

- Max at  $0K$  and decreases continuously to zero as the temp is increased to the  $T_c$ .

- At  $T=T_c$ , all the SC electrons ~~become~~ become normal electrons.

- The gap decreases from a value of about  $3.5 k_B T_c$  at  $0K$  to zero at the  $T_c$ .

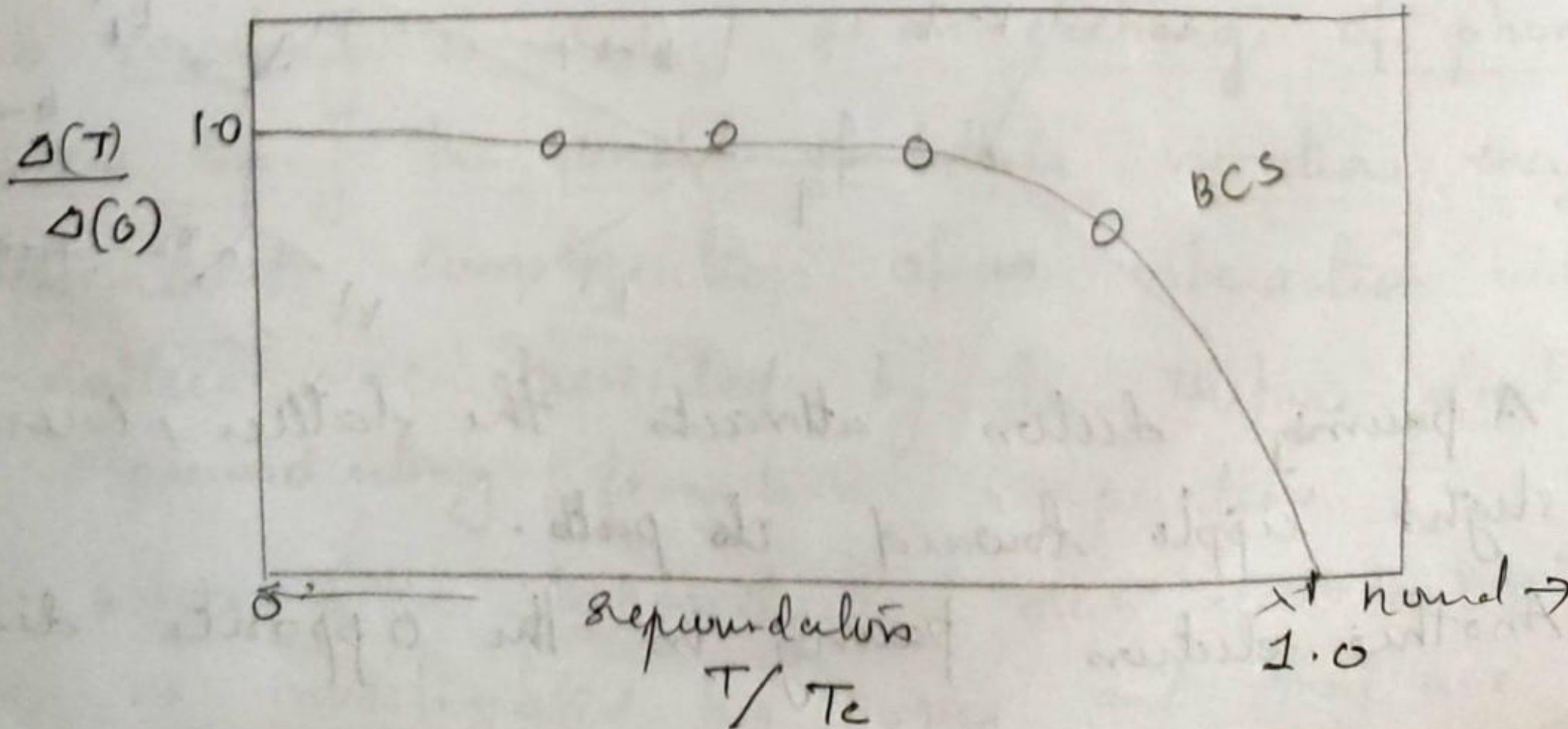
$k_B$  = Boltzmann constant  
 $T_c$  = Critical Temp.

$$E_g = 2\Delta = 2b k_B T_c$$

$$E_g / k_B T_c = 2b$$

$\rightarrow \max T_c < 35K$   
 $= 20K$

$T=T_c \rightarrow \Delta(T)=0$  gap energy = 0





# 8/9/21 Superconductivity - BCS Theory

- from the isotopic effect & energy gap -
- SC due to <sup>change in</sup> lattice property
- crystal structure not changing

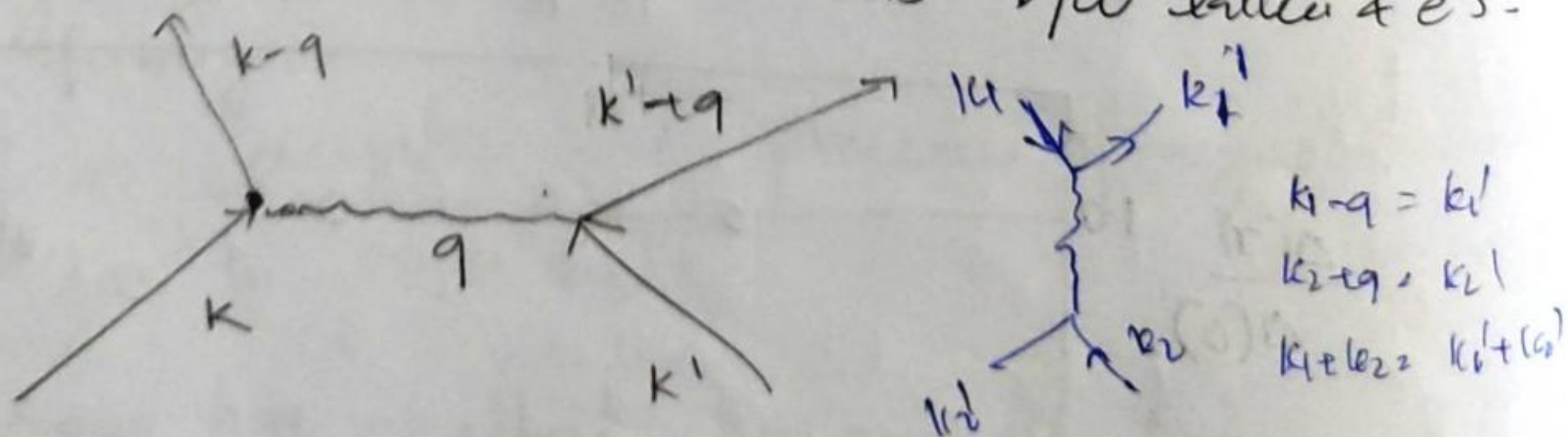
## BCS Theory

- Bardeen, Cooper, Schrieffer attributed the cause of superconductivity to pair of electrons formed by the interactions b/w two  $e^-$ s via an exchange of a phonon.

- This is an electron-phonon-electron interaction which attracts and binds two electrons together forming a pair called a "Cooper pair".

- One electron interacts with a positive ion in the lattice and deforms the lattice, a second electron with compatible momentum (pairing nearby) interacts with the same ion in the distorted lattice so as to minimize its energy.

- (phonon-lattice-interaction) - strong interaction (more resistance)  
 - good conductors - weak interaction b/w lattice &  $e^-$ s



- A pairing electron attracts the lattice, causing a slight ripple toward its path.

- Another electron pairing in the opposite direction



is attracted to that displacement.

Cooper pair formation - electrons will be interacting with the lattice and the lattice will be emitting a phonon, another  $e$  will be interacting with the same lattice will be absorbed by that phonon, so the total momentum will be conserved.

Cooper pairs

- The transition of a metal from the normal to the superconducting state has the nature of a condensation of the electrons into a state which leaves a band gap above them.
- This kind of condensation is seen with super fluid helium, but helium is made up of bosons -- multiple electrons can't collect into a single state because of the Pauli exclusion principle.
- Froehlich was first to suggest that the electrons act as pairs coupled by lattice vibrations in the material.
- This coupling is viewed as an exchange of phonons, phonons being the quanta of lattice vibrations energy.
- Experimental confirmation of an interaction with the lattice was provided by the isotope effect on the superconducting transition temperature.
- The boson-like behaviour of such electron pairs was investigated by Cooper and they are



called Cooper pairs.

- The condensation of Cooper pairs is the foundation of the BCS theory of superconductivity.

(Energy gap - energy needed to break Cooper pair bond)

- coherence length - how long it can move with bond (together).

{ not obey Pauli's exclusion principle, large no. of Cooper pairs can accommodate 1 energy state.

- have integral spins - acts like bosons.

- All will occupy in the BCS ground state. Fermi level is reduced.

- The electrons forming Cooper pairs have equal and opposite momentums one in spin up and the other in spin down state.

- If the state with spin up ( $\uparrow$ ) and  $+k$  is occupied then the corresponding state with down spin ( $\downarrow$ ) and  $-k$  is also occupied. Similarly if the state with spin up ( $\uparrow$ ) and  $+k$  is vacant, then the corresponding state with down spin ( $\downarrow$ ) and  $-k$  is also vacant.

- Net spin of the Cooper pair is zero.

- They condense into a quantum mechanical ground state with a long range order called coherence length.

- Total energy of the system minimizes and a small



energy gap  $\Delta$  is formed near the Fermi energy  $E_f$ ,

$$\Delta \approx 1.4 k_B T_c$$

( $e^-$ s in boson form, many Cooper pairs sharing the same energy level, all try to go to <sup>ground state of</sup> BCS level. Fermi level reduced (position). Gap b/w Fermi level & upper state).

- At  $T = 0$  K energy gap is maximum as pairing is max.  
(Cooper pairs max)

- At  $T = T_c$ , SC and energy gap disappears as all pairing are broken - pairs  $\rightarrow$  normal  $e^-$  (NC).

- Single electrons are scattered by the vibrating ions and experience opposition, hence  $\rho \neq 0$ , but Cooper pairs are not scattered by the vibrating ions, so  $\rho = 0$   
(Cooper pair slide through lattice.)

### Elements of BCS Theory.

- BCS theory of Superconductivity.

(drawback = max transition temp is 135 K at present  
According to BCS theory  $T_c < 20$  K).

BCS - for low temp conductor.)

- The properties of Type 1 Superconductors were modeled successfully by the efforts of John Bardeen, Leon Cooper, and Robert Schrieffer is what is commonly called the BCS theory.

- A key conceptual element in this theory is the pairing of electrons close to the Fermi level into



Cooper pairs through interactions with the crystal lattice.

(~~Cooper~~ electron  $\rightarrow$  Cooper pair, from the surface moving to inside the BCS level, this process continuous). trying to reduce energy).

- This pairing results from a slight attraction b/w the electrons related to lattice vibrations; the coupling to the lattice is called a phonon interaction.

- Pair of electrons can behave very differently from single electrons which are fermions and must obey the Pauli exclusion principle.

- The pair of electrons act now like bosons which can condense into the same energy level.

- The electron pairs have a slightly lower energy and leave an energy gap above them on the order of  $0.001\text{ eV}$  which inhibits the kind of collision interactions which lead to ordinary resistivity.

- For temperatures such that the thermal energy is less than the band gap, the material exhibits zero resistivity.

- B, C, & S received Nobel prize in 1972 for the development of the theory of superconductivity.

- In the normal state of a metal, electrons move independently, whereas in the BCS state, they are



bound into "Cooper pairs" by the attractive interaction. The BCS formalism is based on the reduced potential for the electrons attraction.

- you have to provide energy equal to the energy gap to break apart, to break one pair you have to change energies of all other pairs.
- This is unlike the normal metal, in which the state of an electron can be changed by adding a arbitrarily small amount of energy.
- The energy gap is highest at low temperatures but does not exist at temperatures higher than the transition temperature.
- The BCS Theory gives an expression of how the gap grows with the strength of attractive interaction and density of states.
- The BCS theory gives the expression of the energy gap that depends on the Temp  $T$  and Critical Temp  $T_c$  and is independent of the material:

$$E = 3.52 k_B T_c \left(1 - \left(\frac{T}{T_c}\right)\right)^{1/2}$$

$$k_B = 1.38 \times 10^{-23} \text{ m}^2 \text{ kg s}^{-2} \text{ K}^{-1}$$

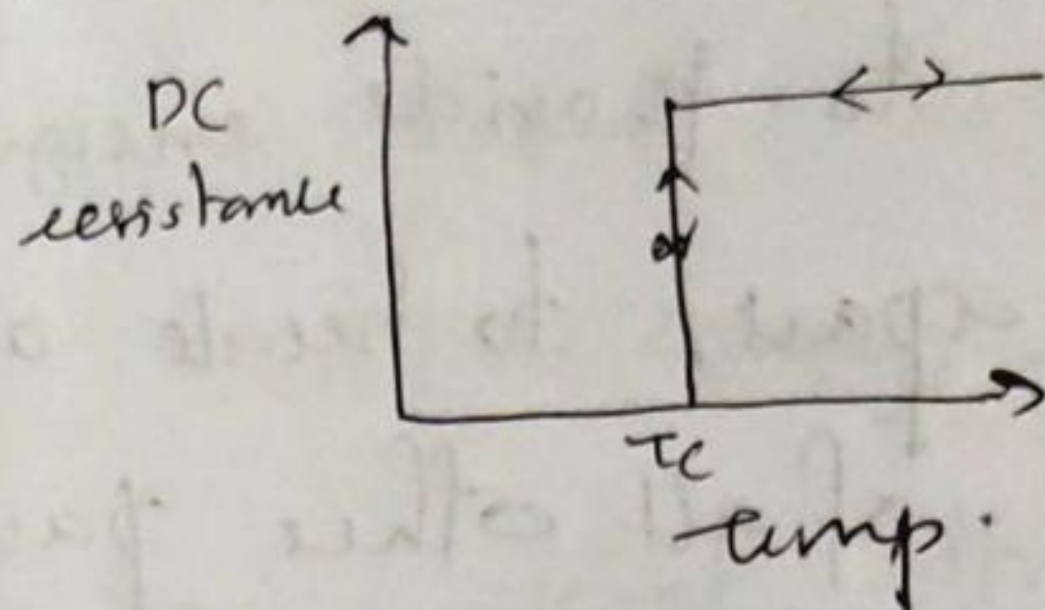


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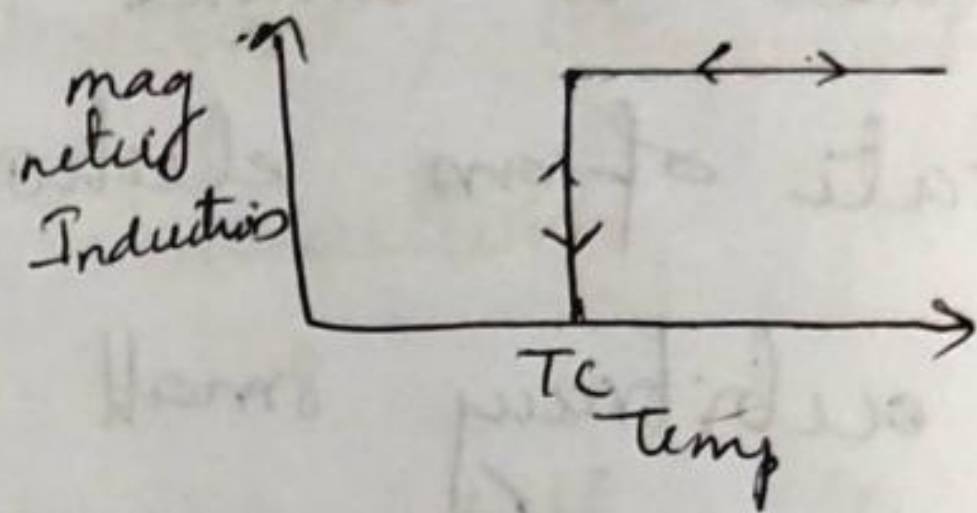
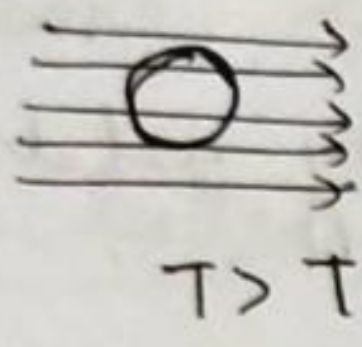
# Flux Quantisation & Tunneling

Three hallmarks of superconductivity

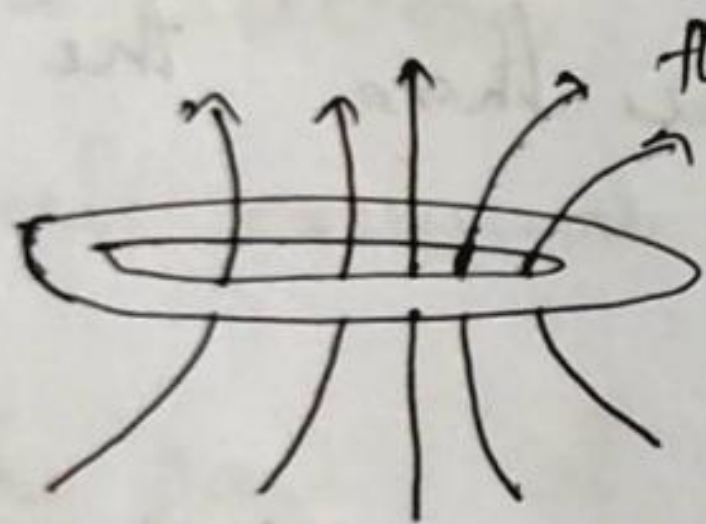
1) Zero resistance



2) complete diamagnetism (Meissner effect)



3) Macroscopic Quantum Effects



flux quantization  $\phi = n\phi_0$

Josephson Effects.

(The material should be in the form of ring or hollow cylinder. The material is normal state and we are applying mag. field. flux line will be everywhere inside outside ring. Once decrease the temp, reach  $T_c$ , the flux lines <sup>on ring</sup> expel out of ring. The flux lines inside the ring is trapped. That flux lines will be quantized).

To prove flux lines are quantised,

let  $\psi(r)$  be the super state wave function.

(Cooper pair is represented by wavefunction  $\psi$ )

$n =$  particle density / number density

$= \psi^* \psi$

(  $n =$  no. of Cooper pairs / unit volume )



$$\psi = \sqrt{n} e^{i\theta(r)} \quad \& \quad \psi^* = \sqrt{n} e^{-i\theta(r)}$$

$\theta =$  phase of wave.

$$\psi^* \psi = n$$

particle density

velocity of particle  $v = \frac{1}{m} (p - \frac{q}{c} A)$

( $p =$  momentum,  $A =$  vector potential)

velocity  $v = \frac{1}{m} \left( \frac{\hbar}{i} \nabla - \frac{q}{c} A \right)$

$$p = m v$$

$$n = \frac{p}{\hbar} - \frac{q}{c} A$$

particle flux:  $\psi^* v \psi$

$$= \frac{n}{m} e^{-i\theta} \left( \frac{\hbar}{i} \nabla - \frac{q}{c} A \right) e^{i\theta} = \frac{n}{m} \left( \frac{\hbar}{i} \nabla e^{i\theta} - \frac{q}{c} A e^{i\theta} \right)$$

$$= \frac{n}{m} \left( \hbar \nabla \theta - \frac{q}{c} A \right)$$

Electric current density:  $j = q \psi^* v \psi$

$$j = \frac{nq}{m} \left( \hbar \nabla \theta - \frac{q}{c} A \right)$$

$$\nabla \times \nabla \theta = 0$$

$$\lambda_L = \left( \frac{m}{nq^2 \mu_0} \right)^{1/2}$$

$$\nabla \times j = \frac{nq}{m} \left( -\frac{q}{c} \nabla \times A \right) = -\frac{nq^2}{mc} B$$

London eq<sup>n</sup> with  $\lambda_L = \sqrt{\frac{mc^2}{4\pi nq^2}}$

According to Meissner effect, mag. field = 0,  $B = 0$

$\therefore$  The mag flux lines cannot penetrate inside the body

Therefore  $j = 0$ . The current will be flowing only on the surface of SC material not inside.

Inside S.C.  $B = j = 0$ ,

$$\Rightarrow j = \frac{nq}{m} \left( \hbar \nabla \theta - \frac{q}{c} A \right) = 0 \Rightarrow \hbar \nabla \theta = \frac{q}{c} A$$



take closed integral on b.s, writ dt

$$\int_C \hbar \nabla \theta \cdot d\mathbf{l} = \frac{q}{c} \int_C \mathbf{A} \cdot d\mathbf{l}$$

$$\hbar \Delta \theta = \frac{q}{c} \int_S \nabla \times \mathbf{A} \cdot d\boldsymbol{\sigma} = \frac{q}{c} \int_S \mathbf{B} \cdot d\boldsymbol{\sigma} \quad (\text{using Stokes law})$$

$$\hbar \Delta \theta = \frac{q}{c} \phi$$

$\phi$  measurable  $\rightarrow$   $\phi$  single valued  $\rightarrow \Delta = 2\pi s$ .

$$\frac{\hbar}{2\pi} \cdot 2\pi s \cdot \theta = \frac{q}{c} \phi$$

$\int \nabla \theta \cdot d\mathbf{l} = 2\pi s$

single value

$$\phi = \frac{hc}{q} s$$

$q =$  total charge of loop pair  $= -2e$

$$(s=1) \quad \phi_0 = \frac{hc}{2e} \approx 2.0678 \times 10^{-7} \text{ gauss cm}^2$$

$=$  fluxoid or fluxon. quantum of flux

flux through ring:  $\phi = \phi_{ext} + \phi_{sc} = s\phi_0$

$\phi_{ext}$  not quantized  $\rightarrow \phi_{sc}$  must adjust

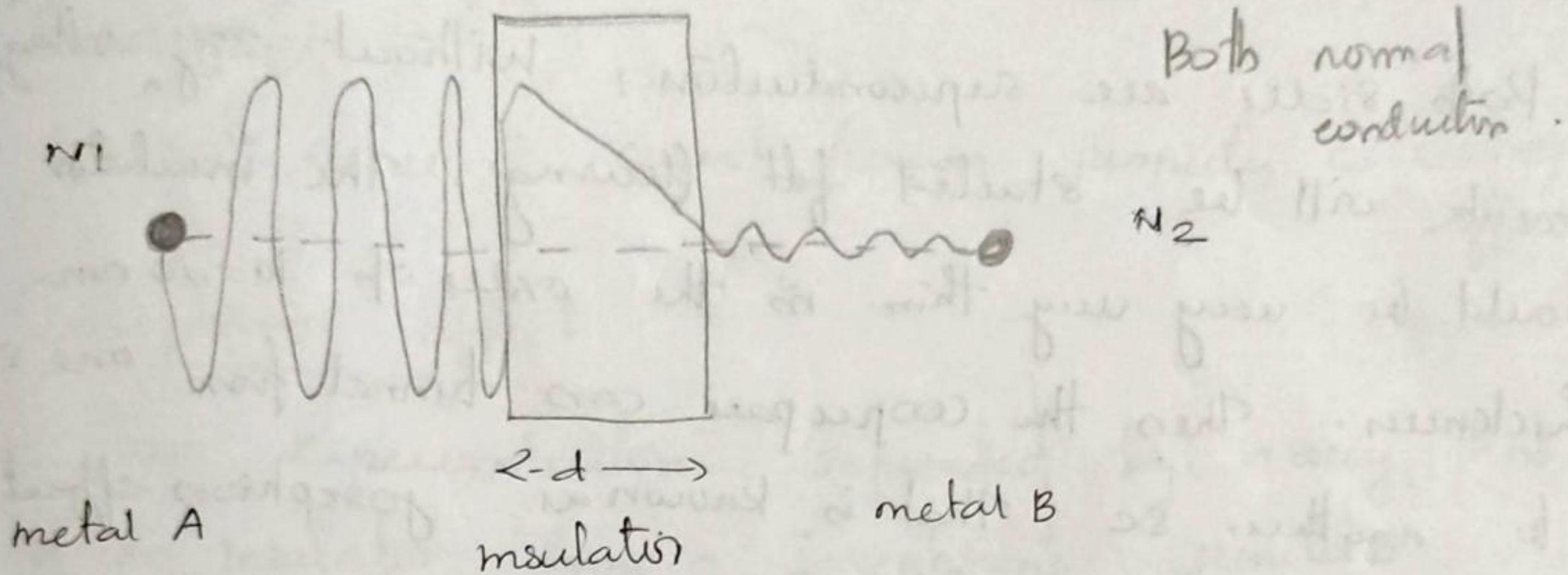
### Tunneling in solid state systems

The award is for their discoveries regarding tunneling phenomena in solids. Half of the prize is divided equally b/w Esaki and Giaever for their experimental discoveries regarding tunneling phenomena in semiconductors and superconductors respectively. The other half is awarded to Josephson for his theoretical predictions of properties

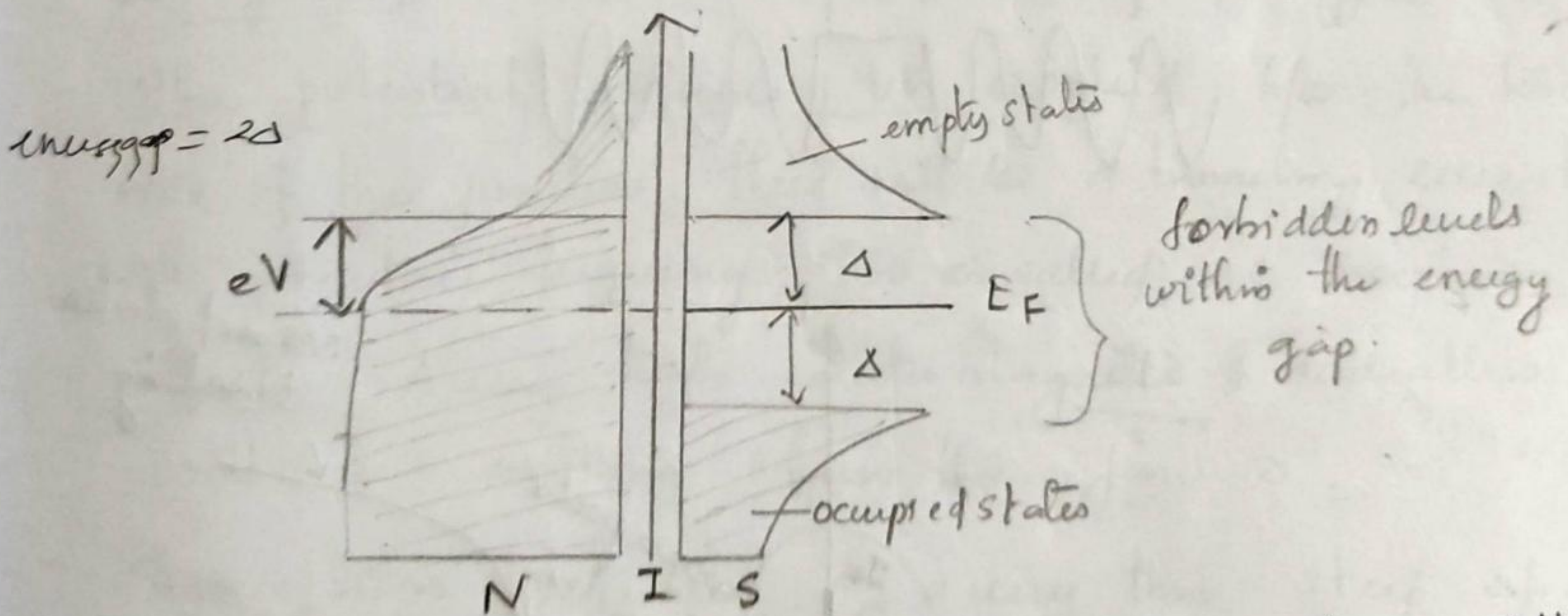


is a supercurrent flowing through a tunnel barrier, in particular the phenomena generally known as the Josephson effects.

## Electron tunneling in solids



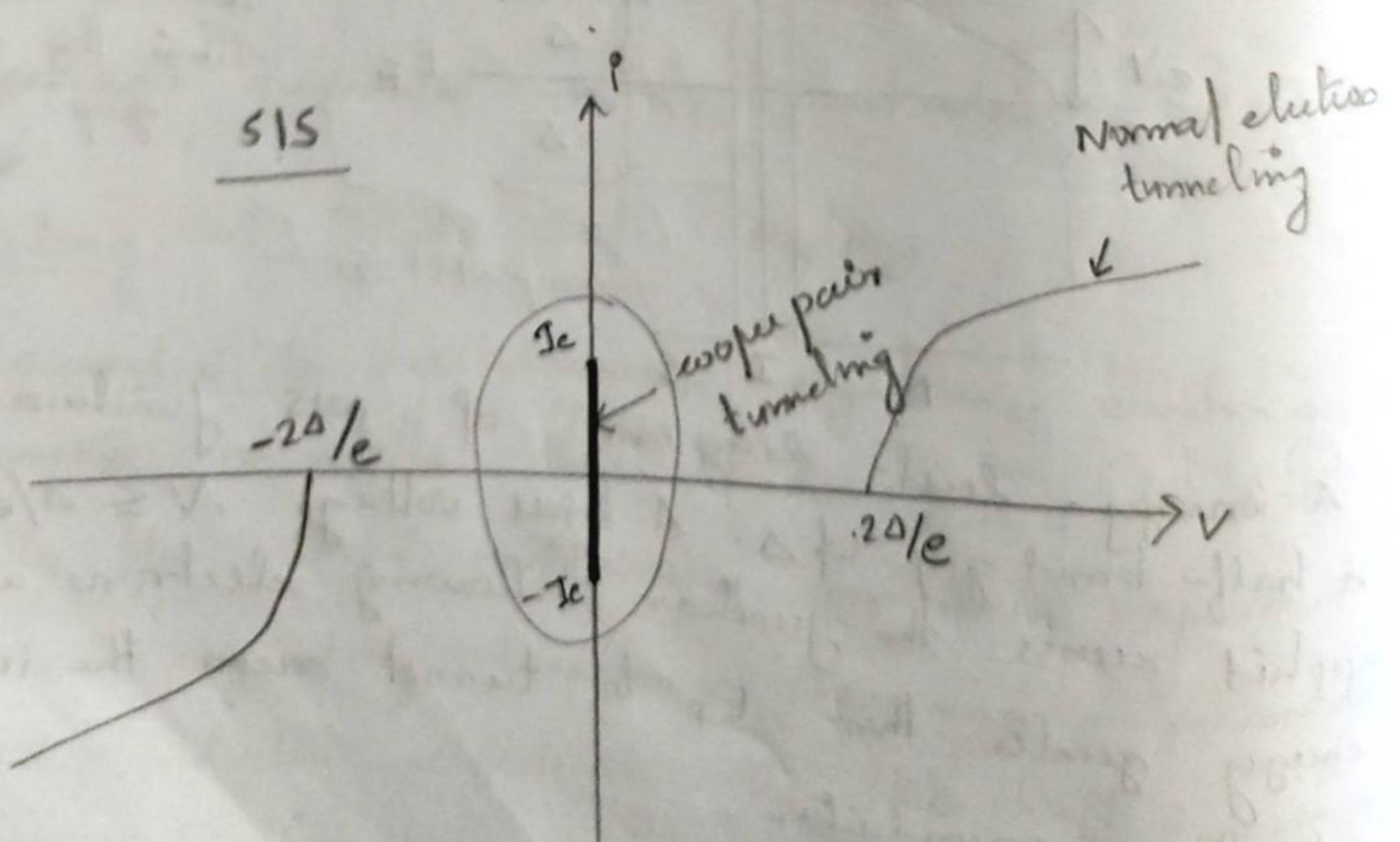
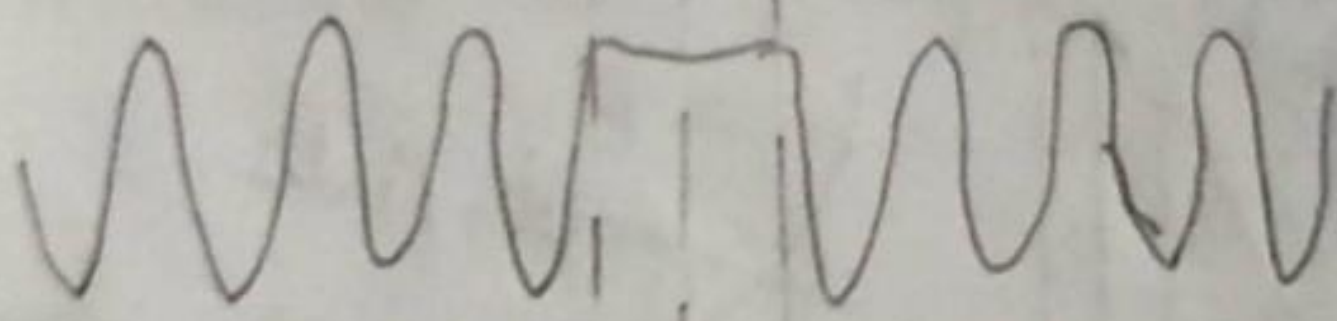
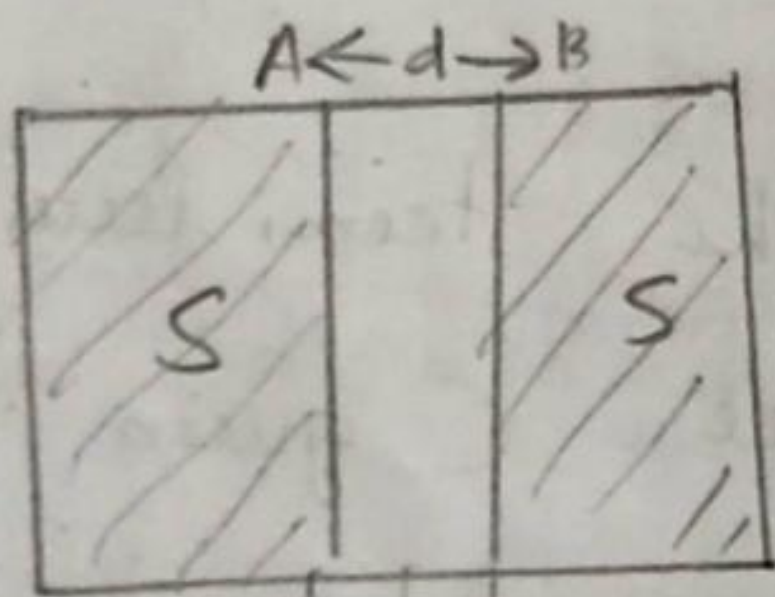
Electrons close to the Fermi level can tunnel from one metal to another. (given insulator is very thin)



A energy-level diagram of a NIS junction with a half-band gap of  $\Delta$ . A bias voltage  $V \leq \Delta/e$  is applied across the junction, allowing electrons with an energy greater than  $E_F$  to tunnel across the insulator to the superconductor.



if Both sides are superconductors, without <sup>internal</sup> applied voltage, current will be started ~~for~~ flowing; The insulator should be very very thin in the order of 10-20 nm. thickness. Then the Cooper pair can tunnel from one SC to another SC. That is known as Josephson effect.





- from the graph, without any <sup>(applied)</sup> external voltage ( $V=0$ ) current is flowing from  $-I_c$  to  $+I_c = DC$  current
- maximum current flowing without applied voltage
- normal  $e^-$  starts tunneling only when voltage is equal to  $\frac{2\Delta}{e}$  energy required to break the Cooper pair.  
( $2\Delta$  gap) - AC
- SIS - super  $e^-$  current flowing property is Josephson effect.

## Josephson Effect

- Two superconductors separated by a very thin strip of an insulator form a Josephson's junction.
- As a consequence of the tunneling of electrons across the insulator <sup>without applied voltage</sup>, there is a net current across the junction. This is called DC Josephson effect.
- If a potential difference  $V$  is applied b/w the two side of the junction, there will be a tunneling current with angular frequency. This is called AC Josephson effect. A very high density magnetic  $\gamma$  radiation will be emitting from the junction.
- Two SCs separated by a very thin strip of an <sup>insulator</sup> ~~insulator~~ forms a Josephson junction.
- The wave nature of moving particles makes electrons to tunnel through the barrier. As a consequence of tunneling of electrons across the insulator there is net current across the junction. This is called d.c Josephson effect. The current flows even in absence of



potential difference.

- The magnitude of current depends on the thickness of the insulators, the nature of the materials, and temperature.

- (superconducting bar - connected with voltmeter shows zero voltage. Because, zero resistance  $V = IR$ ,  $V = 0$ .)

- Break superconducting bar - voltmeter deflects.

14/9/21

### Theory of DC Josephson effect.

Let  $\psi_1$  be the probability amplitude of  $e$  pair on one side of the junction &  $\psi_2$  be the probability <sup>amplitude</sup> on the other side. And assume that both the SCs are identical and suppose both are in zero potential.

The time dependent Sch eq

$$i\hbar \frac{\partial \psi}{\partial t} = H\psi$$

can be applied to the two SCs having amplitude  $\psi_1$  &  $\psi_2$ . As

$$i\hbar \frac{\partial \psi_1}{\partial t} = \cancel{H\psi_1} (H + T) \psi_2 \quad \text{--- (1)}$$

$$i\hbar \frac{\partial \psi_2}{\partial t} = (H + T) \psi_1 \quad \text{--- (2)}$$

where  $T$  represents the effect of electron pair coupling or the transfer interaction across the insulator.

-  $T$  is the measure of leakage of  $\psi_1$  into the region II. and  $\psi_2$  is the region I.

- Suppose the gap b/w the two SC is large, then  $T = 0$ . (Thickness of insulator large)



$$\psi_1 = \sqrt{n_1} e^{i\theta_1} \quad \text{--- (3)}$$

$$\psi_2 = \sqrt{n_2} e^{i\theta_2} \quad \text{--- (4)}$$

$$\delta = \theta_2 - \theta_1$$

where  $n_1$  &  $n_2$  are the number densities of scrs of regions I & II.

Substitute (3) & (4) in (1) & (2).

$$i\hbar \frac{\partial}{\partial t} (\sqrt{n_1} e^{i\theta_1}) = \hbar T \psi_2$$

$$\frac{\partial}{\partial t} (\sqrt{n_1} e^{i\theta_1}) = -i T \psi_2$$

$$\frac{1}{2\sqrt{n_1}} \frac{\partial n_1}{\partial t} e^{i\theta_1} + \sqrt{n_1} i e^{i\theta_1} \frac{\partial \theta_1}{\partial t} = -i T \psi_2$$

$$\frac{1}{2\sqrt{n_1}} \frac{\partial n_1}{\partial t} e^{i\theta_1} + i \psi_1 \frac{\partial \theta_1}{\partial t} = -i T \psi_2 \quad \text{--- (5)}$$

substituting (5) in (2),

$$i\hbar \frac{\partial}{\partial t} (\sqrt{n_2} e^{i\theta_2}) = \hbar T \psi_1$$

$$\frac{\partial}{\partial t} (\sqrt{n_2} e^{i\theta_2}) = -i T \psi_1$$

$$\frac{1}{2\sqrt{n_2}} \frac{\partial n_2}{\partial t} e^{i\theta_2} + \sqrt{n_2} i e^{i\theta_2} \frac{\partial \theta_2}{\partial t} = -i T \psi_1$$

$$\frac{1}{2\sqrt{n_2}} \frac{\partial n_2}{\partial t} e^{i\theta_2} + i \psi_2 \frac{\partial \theta_2}{\partial t} = -i T \psi_1 \quad \text{--- (6)}$$

multiply (6) with  $\sqrt{n_1} e^{-i\theta_1}$

$$\sqrt{n_1} e^{-i\theta_1} \left( \frac{1}{2\sqrt{n_2}} \frac{\partial n_2}{\partial t} e^{i\theta_2} + \sqrt{n_2} i e^{i\theta_2} \frac{\partial \theta_2}{\partial t} \right) = -i T \sqrt{n_2} e^{i\theta_2} \sqrt{n_1} e^{-i\theta_1}$$

$$\frac{1}{2} \frac{\partial n_1}{\partial t} + \sqrt{n_1} e^{i\theta_1} \cdot \sqrt{n_2} i e^{i\theta_1} \frac{\partial \theta_1}{\partial t} = -i T \sqrt{n_1 n_2} e^{i(\theta_2 - \theta_1)}$$

$$\frac{1}{2} \frac{\partial n_1}{\partial t} + i n_1 \frac{\partial \theta_1}{\partial t} = -i T \sqrt{n_1 n_2} e^{i(\theta_2 - \theta_1)} \quad \text{--- (7)}$$

$$\frac{1}{2} \frac{\partial n_1}{\partial t} + i n_1 \frac{\partial \theta_1}{\partial t} = -i T \sqrt{n_1 n_2} (\cos(\theta_2 - \theta_1) + i \sin(\theta_2 - \theta_1))$$

$$\frac{1}{2} \frac{\partial n_1}{\partial t} + i n_1 \frac{\partial \theta_1}{\partial t} = -i T \sqrt{n_1 n_2} \cos \delta + T \sqrt{n_1 n_2} \sin \delta \quad \text{--- (8)}$$



multiply ⑥ with  $\sqrt{n_2} e^{-i\theta_2}$

$$\sqrt{n_2} e^{-i\theta_2} \left( \frac{1}{\alpha \sqrt{n_2}} e^{i\theta_2} \right) \frac{\partial n_2}{\partial t} + (\sqrt{n_2} e^{-i\theta_2}) i \sqrt{n_2} e^{+i\theta_2} \frac{\partial \theta_2}{\partial t} = -iT \sqrt{n_1} e^{i\theta_1} \times \sqrt{n_2} e^{-i\theta_2}$$

$$\begin{aligned} \frac{1}{2} \frac{\partial n_2}{\partial t} + i n_2 \frac{\partial \theta_2}{\partial t} &= -iT \sqrt{n_1 n_2} e^{i(\theta_1 - \theta_2)} \\ &= -iT \sqrt{n_1 n_2} e^{-i(\theta_2 - \theta_1)} \\ &= -iT \sqrt{n_1 n_2} (\cos(\theta_2 - \theta_1) - i \sin(\theta_2 - \theta_1)) \end{aligned}$$

↓

$$\frac{1}{2} \frac{\partial n_2}{\partial t} + i n_2 \frac{\partial \theta_2}{\partial t} = -iT \sqrt{n_1 n_2} \cos \delta - T \sqrt{n_1 n_2} \sin \delta \quad \text{--- ⑧}$$

Separating into real & imaginary parts.

from ⑧ real part,

$$\frac{1}{2} \frac{\partial n_1}{\partial t} = T \sqrt{n_1 n_2} \sin \delta$$

$$\boxed{\frac{\partial n_1}{\partial t} = 2T \sqrt{n_1 n_2} \sin \delta} \quad \text{--- I}$$

imaginary part,

$$n_1 \frac{\partial \theta_1}{\partial t} = -T \sqrt{n_1 n_2} \cos \delta$$

$$\boxed{\frac{\partial \theta_1}{\partial t} = -T \sqrt{\frac{n_2}{n_1}} \cos \delta} \quad \text{--- II}$$

from ⑧ real part,

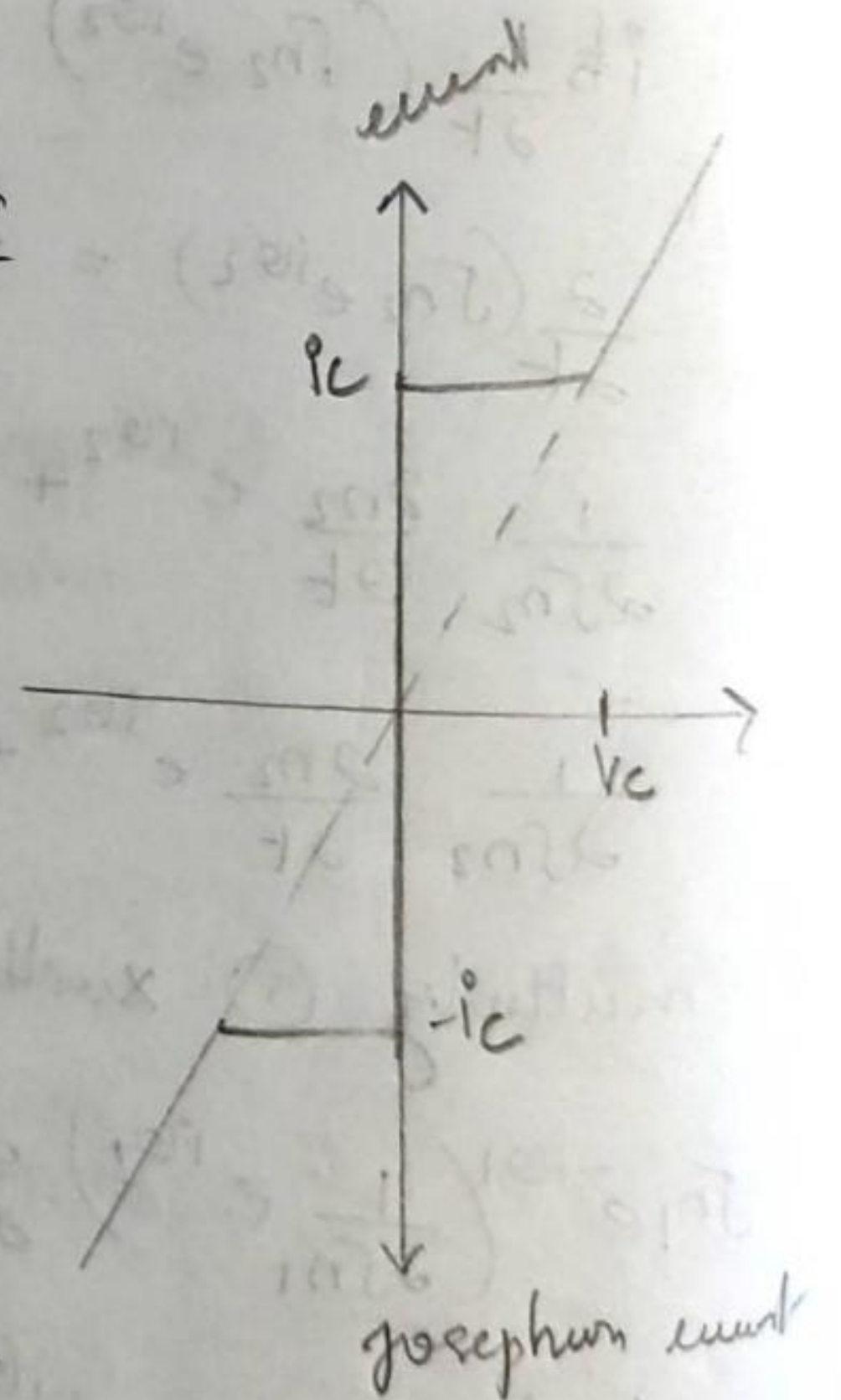
$$\frac{1}{2} \frac{\partial n_2}{\partial t} = -T \sqrt{n_1 n_2} \sin \delta$$

$$\boxed{\frac{\partial n_2}{\partial t} = -2T \sqrt{n_1 n_2} \sin \delta} \quad \text{--- III}$$

imaginary part,

$$n_2 \frac{\partial \theta_2}{\partial t} = -T \sqrt{n_1 n_2} \cos \delta$$

$$\boxed{\frac{\partial \theta_2}{\partial t} = -T \sqrt{\frac{n_1}{n_2}} \cos \delta}$$





for two identical superconductors  $n_1 = n_2$

$$\frac{\partial n_1}{\partial t} = 2Tn \sin \delta$$

$$\frac{\partial n_2}{\partial t} = -2Tn \sin \delta$$

$$\frac{\partial n_2}{\partial t} = -\frac{\partial n_1}{\partial t}$$

current-flow from the junction is proportional to  $\frac{\partial n_2}{\partial t}$  or  $-\frac{\partial n_1}{\partial t}$

current flow  $\propto \frac{\partial n_1}{\partial t}$

$$\frac{\partial \theta_1}{\partial t} = -T \cos \delta$$

$$\frac{\partial \theta_2}{\partial t} = -T \cos \delta$$

$$\frac{\partial \theta_1}{\partial t} = \frac{\partial \theta_2}{\partial t} \Rightarrow \frac{\partial (\theta_2 - \theta_1)}{\partial t} = 0 \quad (\text{phase difference is time independent})$$

$$\frac{\partial \delta}{\partial t} = 0 \Rightarrow \delta = \text{constant}$$

$\therefore$  we can conclude that supercurrent  $J$  of the superconductor <sup>pairs</sup> across the junction depends on the phase difference  $\delta$ .

$$J = J_0 \sin \delta$$

Since  $\delta$  is a constant,  $J$  will be a constant DC current &  $J_0$  is proportional to  $T$  & here we have taken the

voltage applied as zero & the current flowing is a constant current. This implies that without applying any voltage a DC current flowing through the junction with maximum value of  $-J_0$  to  $+J_0$ . Depending on the value of  $\theta_2 - \theta_1$  and this effect is known as DC Josephson effect.

So plotting a graph current density vs  $\theta$ .

$$+J_0 \text{ at } \sin 90$$

$$-J_0 \text{ at } \sin 270$$

- no phase change in DC J effect. (

T = Transfer interaction.



## AC Josephson effect

Let a voltage  $V$  is applied to the Josephson junction, an  $e^-$  pair experience a potential difference  $qV$  on passing across the junction where  $q = -2e$ . [because Cooper pair consists of  $2e^-$ ] instead of  $e^-$  tunneling, Cooper pair is tunneling.

So we can say that a pair on one side has a potential of  $-eV$  and a pair on the other side has a potential of  $+eV$ . So that potential difference will become  $-2eV$ . So the eq<sup>n</sup> of motion become.

$$i\hbar \frac{\partial \psi_1}{\partial t} = \hbar T \psi_2 - eV \psi_1 \quad \text{--- (1)}$$

$$i\hbar \frac{\partial \psi_2}{\partial t} = \hbar T \psi_1 + eV \psi_2 \quad \text{--- (2)}$$

$$\psi_1 = \sqrt{n_1} e^{i\theta_1}$$

$$\psi_2 = \sqrt{n_2} e^{i\theta_2}$$

$\delta = \theta_2 - \theta_1$   
phase difference.

$$(1) \Rightarrow i\hbar \frac{\partial (\sqrt{n_1} e^{i\theta_1})}{\partial t} = \hbar T \sqrt{n_2} e^{i\theta_2} - eV \sqrt{n_1} e^{i\theta_1}$$

$$\left( \frac{i\hbar \sqrt{n_1}}{2\sqrt{n_1}} \frac{\partial n_1}{\partial t} e^{i\theta_1} + \sqrt{n_1} e^{i\theta_1} \cdot \frac{\partial \theta_1}{\partial t} \right) = \hbar T \sqrt{n_2} e^{i\theta_2} - eV \sqrt{n_1} e^{i\theta_1} \quad \text{--- (3)}$$

$$\text{--- (3)} \times \sqrt{n_1} e^{-i\theta_1}$$

$$i\hbar \frac{\sqrt{n_1}}{2\sqrt{n_1}} e^{-i\theta_1} \frac{\partial n_1}{\partial t} e^{i\theta_1} + \sqrt{n_1} e^{i\theta_1} e^{-i\theta_1} \frac{\partial \theta_1}{\partial t} = \hbar T \sqrt{n_1} \sqrt{n_2} e^{i\theta_2} e^{-i\theta_1} - eV \sqrt{n_1} \sqrt{n_1} e^{i\theta_1} e^{-i\theta_1}$$

$$= \hbar T \sqrt{n_1} \sqrt{n_2} e^{i(\theta_2 - \theta_1)} - eV n_1$$

$$i\hbar \frac{1}{2} \frac{\partial n_1}{\partial t} + \hbar n_1 \frac{\partial \theta_1}{\partial t} = \hbar T \sqrt{n_1} \sqrt{n_2} e^{i(\theta_2 - \theta_1)} - eV n_1$$



$$i\hbar \frac{1}{2} \frac{\partial n_1}{\partial t} - \hbar n_1 \frac{\partial \theta_1}{\partial t} = \hbar T \sqrt{n_1 n_2} e^{i\delta} - eV \sqrt{n_1 n_2}$$

$$i\hbar \frac{1}{2} \frac{\partial n_1}{\partial t} - \hbar n_1 \frac{\partial \theta_1}{\partial t} = \hbar T \sqrt{n_1 n_2} (\cos \delta + i \sin \delta) - eV \sqrt{n_1 n_2}$$

$$i\hbar \frac{1}{2} \frac{\partial n_1}{\partial t} - \hbar n_1 \frac{\partial \theta_1}{\partial t} = \hbar T \sqrt{n_1 n_2} \cos \delta + i \hbar T \sqrt{n_1 n_2} \sin \delta - eV \sqrt{n_1 n_2}$$

Real part

$$-\hbar n_1 \frac{\partial \theta_1}{\partial t} = \hbar T \sqrt{n_1 n_2} \cos \delta - eV \sqrt{n_1 n_2}$$

$$-\hbar \frac{\partial \theta_1}{\partial t} = \hbar T \frac{\sqrt{n_2}}{\sqrt{n_1}} \cos \delta - eV \frac{\sqrt{n_2}}{\sqrt{n_1}}$$

$$\boxed{\frac{\partial \theta_1}{\partial t} = -T \frac{\sqrt{n_2}}{\sqrt{n_1}} \cos \delta + \frac{eV}{\hbar}} \quad \text{--- I.}$$

Imaginary part

$$\hbar \frac{1}{2} \frac{\partial n_1}{\partial t} = \hbar T \sqrt{n_1 n_2} \sin \delta$$

$$\boxed{\frac{\partial n_1}{\partial t} = 2T \sqrt{n_1 n_2} \sin \delta} \quad \text{--- II.}$$

$$\textcircled{2} \Rightarrow i\hbar \frac{\partial \sqrt{n_2} e^{i\theta_2}}{\partial t} = \hbar T \sqrt{n_1} e^{i\theta_1} + eV \sqrt{n_2} e^{i\theta_2}$$

$$\left( i\hbar \left( e^{i\theta_2} \frac{1}{2\sqrt{n_2}} \frac{\partial n_2}{\partial t} + \sqrt{n_2} \cdot i e^{i\theta_2} \cdot \frac{\partial \theta_2}{\partial t} \right) \right)$$

$$= \hbar T \sqrt{n_1} e^{i\theta_1} + eV \sqrt{n_2} e^{i\theta_2} \Big) \times \sqrt{n_2} e^{-i\theta_2}$$

$$= i\hbar \frac{\sqrt{n_2}}{2\sqrt{n_2}} e^{-i\theta_2} \cdot e^{i\theta_2} \frac{\partial n_2}{\partial t} + i \hbar \sqrt{n_2} \sqrt{n_2} e^{-i\theta_2} e^{i\theta_2} \frac{\partial \theta_2}{\partial t}$$

$$= \hbar T \sqrt{n_1 n_2} e^{-i(\theta_2 - \theta_1)} + eV \sqrt{n_2} \sqrt{n_2} e^{i\theta_2 - i\theta_2}$$



$$\frac{i\hbar}{2} \frac{dn_2}{dt} - \hbar n_2 \frac{d\theta_2}{dt} = \hbar T \sqrt{n_1 n_2} e^{-i\theta} + eV n_2$$

$$\frac{i\hbar}{2} \frac{dn_2}{dt} - \hbar n_2 \frac{d\theta_2}{dt} = \hbar T \sqrt{n_1 n_2} (\cos\theta - i\sin\theta) + eV n_2$$

Real part,

$$-\hbar n_2 \frac{d\theta_2}{dt} = \hbar T \sqrt{n_1 n_2} \cos\theta + eV n_2$$

$$\frac{d\theta_2}{dt} = -T \frac{\sqrt{n_1 n_2} \cos\theta}{n_2} - \frac{eV}{\hbar}$$

$$\boxed{\frac{d\theta_2}{dt} = -T \sqrt{\frac{n_1}{n_2}} \cos\theta - \frac{eV}{\hbar}} \quad \text{--- III}$$

Imaginary part

$$\frac{\hbar}{2} \frac{dn_2}{dt} = -\hbar T \sqrt{n_1 n_2} \sin\theta$$

$$\boxed{\frac{dn_2}{dt} = -2T \sqrt{n_1 n_2} \sin\theta} \quad \text{--- IV}$$

$$\frac{dn_1}{dt} = 2T \sqrt{n_1 n_2} \sin\theta$$

$$\frac{d\theta_1}{dt} = -T \sqrt{\frac{n_2}{n_1}} \cos\theta + \frac{eV}{\hbar}$$

$$\frac{dn_2}{dt} = -2T \sqrt{n_1 n_2} \sin\theta$$

$$\frac{d\theta_2}{dt} = -T \sqrt{\frac{n_1}{n_2}} \cos\theta - \frac{eV}{\hbar}$$



for identical sides  $n_1 = n_2$

$$\frac{dn_1}{dt} = 2Tn \sin \delta$$

$$\frac{dn_2}{dt} = -2Tn \sin \delta$$

$$\frac{dn_2}{dt} = -\frac{dn_1}{dt}$$

$$\frac{d\theta_1}{dt} = -T \cos \delta + \frac{eV}{\hbar}$$

$$\frac{d\theta_2}{dt} = -T \cos \delta - \frac{eV}{\hbar}$$

$$\frac{d\theta_2}{dt} - \frac{d\theta_1}{dt} = -\frac{2eV}{\hbar}$$

$$\frac{d(\theta_2 - \theta_1)}{dt} = -\frac{2eV}{\hbar}$$

$$\frac{d\delta}{dt} = -\frac{2eV}{\hbar} \quad \Rightarrow \quad \int_0^t d\delta = \int_0^t -\frac{2eV}{\hbar} dt$$

$$\delta(0) = \delta(0) = ($$

$$\delta(t) - \delta(0) = -\frac{2eV}{\hbar} t$$

$$\delta(t) = \delta(0) - \frac{2eV}{\hbar} t \quad (\delta = \text{time dependent})$$

$$J = J_0 \sin \delta = J_0 \sin \left( \delta(0) - \frac{2eV}{\hbar} t \right)$$

element oscillates with frequency (A very high electromag- netic radiation will be emitted with freq  $\omega$ )

$$\boxed{\omega = \frac{2eV}{\hbar}} \Rightarrow \hbar\omega = 2eV$$

This is the ac Josephson effect. A dc voltage of 1 eV produces a frequency of 483.6 MHz. The relation says that a photon of energy  $\hbar\omega = 2eV$  is emitted or absorbed when an electron pair crosses



the barrier.  
 - To be used for a precise measurement of  $h/e$

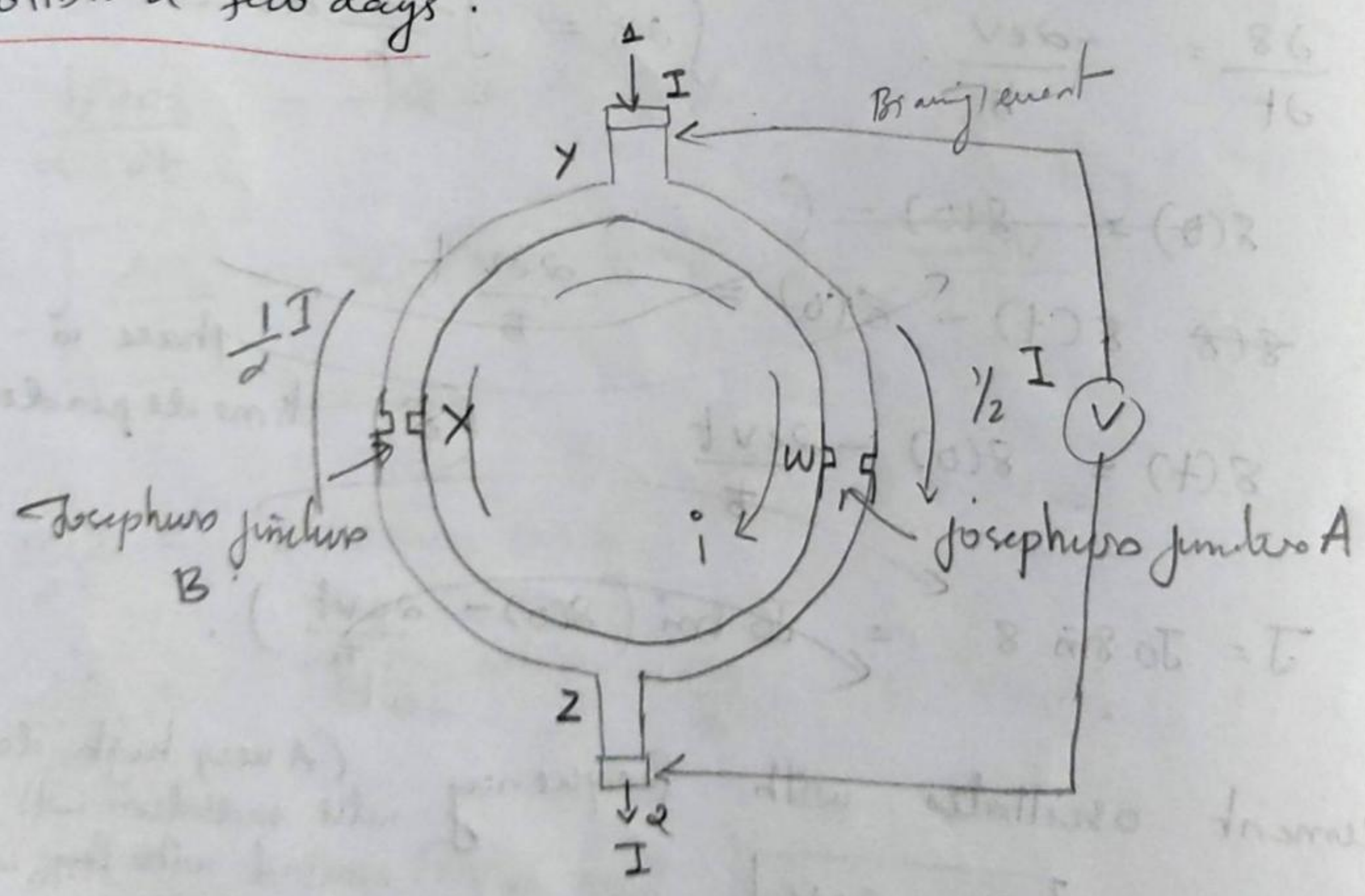
## SQUIDS

Superconducting Quantum Interference Device

- They are type of extremely sensitive magnetometer that contains a Josephson junction.

(two sets - closed one - two shells joining positions is insulator (junction))

- They are so sensitive that they can detect a field change of  $5 \times 10^{-14}$  gauss ( $1/10^{13}$  of the earth's mag field) within a few days.



(There will no effect of external mag field inside squid)

### Principle

phase change due to mag field  $\rightarrow$  current flows  $\rightarrow$  voltage change.



- DC Squids are a current loop with two Josephson junctions in it that will carry a current of  $\frac{I}{2}$  in each branch in the absence of an external magnetic field.

- In the presence of a field there is a secondary current produced that means the current is one branch and decreases the current of another branch.

- This will continue until there is a critical current reached that will then cause the magnetic field to switch and thus the current to change.

- (~~Joseph~~ - a voltage will be appearing on the Josephson junctions

- One branch exceeding critical ~~near~~ current means it is a normal conductor, flux will be started penetrating inside the body.) ~~phase change occurs~~

(one period of voltage variation corresponds to an increase of one flux quantum).

Let the phase difference b/w points 1 and 2 taken on a path through junctions  $A(B)$  be  $\delta A(\delta B)$ .

when no external field is applied,  $\phi = 0$

$$\delta A = \delta B$$

when ext field applied  $\phi \neq 0 \Rightarrow \delta B - \delta A = \frac{2e}{\hbar c} \phi$

The phase difference around a closed circuit which encompasses a total magnetic flux  $\phi$ .

$$\delta_B = \delta_0 + \frac{e}{\hbar c} \phi, \quad \delta_A = \delta_0 - \frac{e}{\hbar c} \phi$$

*They differ*



The total current.

$$I_{\text{total}} = I_c \sin \phi_0$$

$$I_{\text{total}} = I_A + I_B$$

$$= I_c \left[ \sin \left( \phi_0 + \frac{e}{\hbar} \phi \right) + \sin \left( \phi_0 - \frac{e}{\hbar} \phi \right) \right]$$

$$= 2 I_c \sin \phi_0 \cos \left( \frac{e}{\hbar} \phi \right)$$

$$I_{\text{total}} = 2 (I_c \sin \phi_0) \cos \frac{e\phi}{\hbar}$$

The current varies with  $\phi$  and has maxima when

$$\frac{e}{\hbar} \phi = s\pi, \quad s = \text{integer.}$$

1. A superconducting <sup>SQUID</sup> quantum interference device is a mechanism used to measure extremely weak signals.
2. Super current changes periodically with magnetic flux.
3. Junction regions have much lower value than the rest of the superconducting ring.
4. When the current in the junction exceeds the critical value, the junctions become normal, then the fluxons penetrate through the link.
5. So the current falls to critical value and then the link reverts to SC ring state.
6. Then the junctions act as gates.

### Applications

- Type II SCs used as very powerful magnet. used in hospitals for NMR. nuclear magnetic resonance.
- maglev train - magnetic levitation train.



& magnet about scr levitate.

scr leg. trans magnet force applied.

- starting ignition - initial trigger.

to stop. use force. no external energy is required.

16/9/21

## High Temperature Superconductors

- based on the Temp which it showing SC tivity.

- liquify the materials.

- almost HT are Type II scrs.

- 1911 1st scrs transition temp was 4K.

- To reduce T - we can use oxygen - difficult.

- need liquified gas He - 4K.

Till 1986 - scr near 20K. used liq He

In 1986 - separate scrs discovered,  $T_c > 77K$ .

- liq Nitrogen - can be used - cheap.  
for cooling

- discovered by Bednorz & Muller. = Ba-La-Cu-O.

## HTSC

- High temperature superconductors have high  $T_c$  values.

- They are not metal or intermetallic compounds but oxides of copper in combination with other elements.

(notations -)

- In 1983, 1987 & 1988, material with  $T_c$  up to 40K, 93K, and 125K have been discovered respectively.

- The HTSC compounds are represented by simplified notations as 1212, 1234 etc. These notations are based on



number of atoms in each metal element.

- they are brittle and easy to form wires and tapes.

(forms of powder - convert to pellet shape or putting powder in silver tube - elongate silver tube to long wire being by heating, silver do not destroy by heating setting properly).

- HTSC wires / tapes provide transmission of electrical power over a long distance without any resistive loss.

- copper oxygen layer is reason for SC truly.

eg:	compound		$T_c$ (K)	
	$YBa_2Cu_3O_{7-8}$	Y-123	92	(yttrium)
	$Bi_2Sr_2CaCu_2O_8$	Bi-2212	84	
	$Bi_2Sr_2Ca_2Cu_3O_{10}$	Bi-2223	110	
	$TlBa_2Ca_2Cu_3O_{10}$	Tl-1223	125	
	$HgBa_2Ca_2Cu_3O_{10}$	Hg1223	138	(mercury)

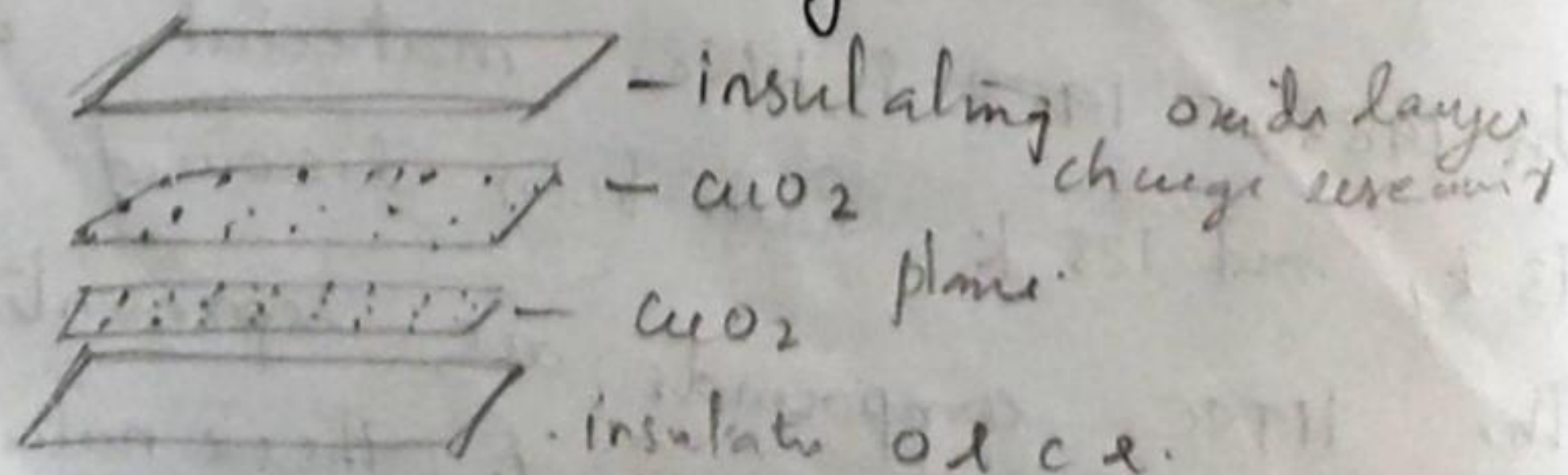
### Pearskite structure

- ceramic - layered pearskite material

$CuO_2$  - planes responsible for superconductivity

- properties show high anisotropy.

$CuO_2$  - plane -





- They are Type II SCs with high  $T_c$  value.

## cuprates

- The resistivity in the normal state varies linearly with temperature.
- Vanishingly small isotope effect ( $\alpha = 0 - 0.2$ ) is considered as important evidence of non-phononicity.
- Observed energy gap is very large, nearly 20-30 meV.
- $\Delta(0)/k_B T_c = 3$  to 4, which is greater than BCS estimated value (1.764)
- The Hall coefficient is temperature dependent.
- An inverted parabolic relation b/w  $T_c$  and hole concentration (holes are responsible for HTSC)
- cuprate SCs are generally considered to be quasi-2D materials with their key properties determined by electrons moving within weakly coupled Copper-oxide ( $\text{CuO}_2$ ) layers.
- Neighboring layers containing ions such as lanthanum, barium, strontium, or other atoms act to stabilize the structure and dope  $e^-$ s or holes onto the copper-oxide layers.
- The cuprate SCs adopt a Perovskite structure.
- The  $\text{CuO}_2$  planes are checkerboard lattices with squares of  $\text{O}_2^-$  ions with  $\text{Cu}^{2+}$  ion at the center of each

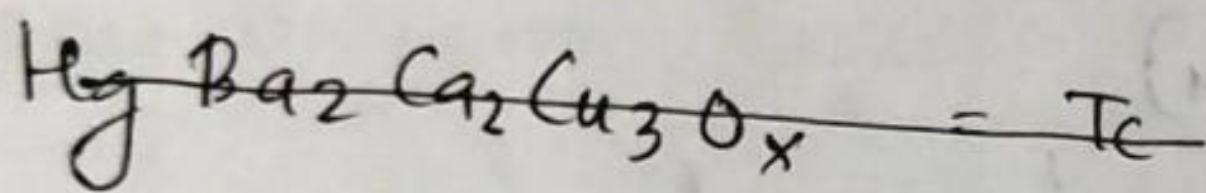


square. chemical formulae of superconducting materials generally contain fractional numbers to describe the doping required for SCing.

- There are several families of cuprate SCs and they can be categorized by the elements they contain and the number of adjacent copper-oxide layers in each SCing block

SCing block

- eg YBCO, BSCCO  
Y123 (Bi2201, Bi2212, Bi2223)



- max T<sub>c</sub> = HgBa<sub>2</sub>Cu<sub>3</sub>O<sub>x</sub> = 133 K.

high Pressure 165 K.

### Iron-based SCs

- IB SCs contain layers of iron & a pnicogen such as arsenic or phosphorus or a chalcogen.
- This is essentially the family with the 2nd highest critical temp behind cuprates.
- most undoped iron-based SCs show a tetragonal orthorhombic structural phase transition followed at lower temperature by magnetic ordering, similar to cuprate SCs
- poor metals
- LaFePO - 4K, LaFeAs - 43K (under pressure)
- LnFeAs - 56K



## MgB<sub>2</sub>

- Magnesium diboride is occasionally referred to as a HTSC because its  $T_c \approx 39\text{K}$ .
- Fullerene SCs where alkali metals are intercalated into C<sub>60</sub> molecules show SCivity at temp of up to 38K.
- Some organic SCs and heavy fermion compounds are considered to be high-temperature SCs because of their high  $T_c$  values relative to their Fermi energy, despite the  $T_c$  values being lower than for many conventional SCs.